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Analogical reasoning based on geometric material in blind pupils

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Analogical reasoning constitutes one of the ways the blind get to know the world; it enriches information and influences thinking.
The purpose of the article is the presentation of results of research on reasoning using geometric analogies in blind pupils aged 10, 12 and 14. The study encompassed a group of 63 blind pupils and 63 seeing pupils. The study used the twelve *series B matrices* from the *Progressive Matrices* of John C. Raven. A detailed analysis of the dependencies between the variables permitted the determination of differences between groups of pupils in terms of reasoning using geometric analogies.

KEY WORDS: analogical reasoning, blind pupils, geometric analogies

Introduction

Analogy is a term used in various daily situations. Would one be forbidden to use analogies, then most probably the would not be able to cope with acquiring new knowledge.

The term „analogy" is of Greek origin, as αναλογια. It is made up of the adverb "*an*", "*ana*" (αν, ανα) and refers to the level of multiplication or repetition, e. g. twice, thrice. The second component of the term is the noun "*logos*" (λογοξ), derived from the verb "*legin*" (λεεγειν), meaning to "*put together*", "*speak*". Etymologically speaking, an "analogy" is a condition referring to the existence of a specific item, or speaking of it.

Zdzisław Chlewiński, Andrzej Falkowski and Piotr Francuz[1] had conducted an analysis of object literature on the various modes of use of the term "*analogy*". Two approaches to defining this term are known. The first is the structural, or syntactic, approach, assuming that an analogy is something equalising, balancing, between two objects or events. It permits an analysis of the relations between selected components of a specific event. Such an analysis can refer to components available by way of perception or constituents of a specific event that are not available directly to our cognition. The second, functional, or pragmatic, approach, explains "analogies" somewhat differently. In view of the proponents of this approach, an analogy refers primarily to seeking relationships between external components, transgressing the scope concerning the event being studied. The process of formation of analogies, or the process of comparison of events, is possible when the readiness emerges to think in cause-and-effect categories.

In the opinion of Edward Nęcka, Jarosław Orzechowski and Błażej Szymura[2], an analogy is a certain transfer between various areas of science. Such a transfer can be positive (if an individual expands their knowledge and uses various techniques to solve the problem at hand) or negative (in this case the use of similar techniques in comparable problem situations is of little effectiveness).

An analogy is the basis for drawing certain conclusions, for fusing data into individual subclasses. Thanks to processes of analogy, it is possible, on the basis of already owned information about a specific object, to assign its properties to other objects. An analogy

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[1] Z. Chlewiński, A. Falkowski, P. Francuz, *Wnioskowanie przez analogię w procesach kategoryzacji*, Wydawnictwo Towarzystwo Naukowe KUL, Lublin 1995, pp. 25–38.

[2] E. Nęcka, J. Orzechowski, B. Szymura, *Psychologia poznawcza,* Wydawnictwo Naukowe PWN, Warszawa 2006, pp. 474–478.

may refer to semiotics and methodology. From the point of view of methodology, an analogy is evaluated as the basic theory constituting the vantage point for certain modes of reasoning or model theory[3].

It must be stressed that the terms “analogy” and „analogical reasoning” are not unequivocal terms.

Analogical reasoning, as Tadeusz Kwiatkowski[4] indicates “is reasoning, in which, if one would have at their disposal in one observed case (or in more cases) a specific set of components, and a fragment of such a set in another, one may come to a conclusion on how this component fits in with the entire set”.

Adam Biela[5], in turn, indicates three main purposes of analogical reasoning:

1) Analogical reasoning can be used to enrich the knowledge that one already has.
2) Analogical reasoning permits the determination of relations between elements.
3) Analogical reasoning increases the probability of a certain conclusion being correct.

Analogical reasoning in blind pupils

Studies on analogical reasoning in pupils with eyesight disabilities are comparably scarce. Bogdan Pietrulewicz[6] had conducted experiments among 60 blind persons in school age (3rd, 5th and 7th

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[3] J. Przybyłowski, *O pewnej interpretacji wnioskowania przez analogie,* [in:] *Logiczne podstawy rozumowań, part II*, ed. by L. Kostro, J. Przybyłowski, Wydawnictwo, Uniwersytet Gdański, Gdańsk 1997, pp. 28–39.

[4] T. Kwiatkowski, *Logika ogólna,* Wydawnictwo UMCS, Lublin 1992, pp. 303.

[5] A. Biela, *Psychologiczne podstawy wnioskowania przez analogię,* Wydawnictwo PWN, Warszawa 1989, pp. 19–39.

[6] B. Pietrulewicz, *Rozwój rozumowanie przez analogię u dzieci niewidomych,* Wydawnictwo PAN, Komitet Nauk Psychologicznych, Zakład Narodowy Ossolińskich, Wrocław, Kraków, Gdańsk, Łódź 1983, pp. 82–95.

grade of primary school). He made the assumption that blind persons, using analogies from an early age, obtain new abilities of its steadily improving use even in situations that are entirely new to them. Experiments by Bogdan Pietrulewicz applied to analogies based on semantic material (part-whole relationships, oppositions, cause-and-effect relationships), using numerical material and geometric analogies. The main hypothesis was the statement that the ability of analogical reasoning develops to the same extent as it does in seeing persons. In terms of analogies based on geometric material, seeing persons tend to achieve better results, doing tasks in terms of visual perception, with the blind performing the same tasks as the seeing group, but by touch. The lowest scores were achieved by blind pupils solving the same problems by touch.

Zofia Sękowska[7] in her studies on analogical reasoning stressed just how great the importance of analogies is, especially in terms of appreciation of external characteristics of specific objects such as: size, colour, shape. These are properties unavailable to the blind. In view of Zofia Sękowska, the blind use analogies when getting to know simple items that are not very complex, and this is possible thanks to them having general information on that specific subject. The blind use analogies as to the size of objects relatively rarely. Another advantage of using analogies is applying them to naming sensory or emotional stimuli and to create surrogate representations. In view of M. Grzegorzewska[8], surrogate representations are *specific substitutes of those parts of a view that are not or not fully available to blind persons and play an important role in shaping their world of images and concepts*. An analogy permits the fusion of stimuli, seeking common properties. Such cognitive activity enriches one's knowledge, expands the scope of surrogate representations that thus become multimodal.

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[7] Z. Sękowska, *Kształcenie dzieci niewidomych,* Wydawnictwo PWN, Warszawa 1974, pp. 175–183.

[8] M. Grzegorzewska *Struktura wyobrażeń surogatowych u niewidomych*, Polskie Archiwum Psychologii, 1927, vol. 1, 4, p. 302.

In order for the blind to be able to make analogies, the ability is necessary to create specific mental representations that emerge as a result of a complex cognitive process based on sensations from various sources: sensory channels (touch, hearing, senses) as well as spatial sensations[9]. These sensations are then transferred to working memory, where they are further processed. In view of Serge Bouaziz, Sandrine Russier and Annie Magnan[10]; Cesar Cornoldi, Maria-Chiara Fastame and Tomaso Vecchi[11]; Morton A. Heller and Edouard Gentaz[12], innate blindness does not hinder the creation of mental images, but they span less information and emerge more slowly than in case of persons utilising eyesight.

The comparison of geometric analogies requires the ability to perform rotations. In order to do them, a reference point is needed, with respect to which the rotation is to take place. Hence, in view of Anna Sfard, the use of various fixed frames in the process of learning things, in which objects, items, drawings can be rotated, whereby curves and diagrams can be created[13]. Brigotte Röder and Frank Rösler[14] have in turn stated that the formation of rotated objects in the imagination of the blind occurs more slowly and contains more errors than is the case of seeing persons.

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[9] C. Cornoldi, M.-C. Fastame, T. Vecchi, *Congenitally blindness and spatial mental imagery*, [in:] *Touching for Knowing*, eds. Y. Hatwell, A. Streri, E. Gentaz, Amsterdam/Philadelphia: John Benjamins Publishing Company, 2003, vol. 53, pp. 173–187.

[10] S. Bouaziz, S., Russier, A. Magnan, *The Copying of Complex Geometric Drawings by Sighted and Visully Impaired Children*, "Journal of Visual Impairment and Blindness" 2005, vol. 99(12), pp. 765–774

[11] C. Cornoldi, M.-C. Fastame, T. Vecchi, *Congenitally blindness and spatial mental imagery*, [in:] *Touching for Knowing*, eds. Y. Hatwell, A. Streri, E. Gentaz, Amsterdam/Philadelphia: John Benjamins Publishing Company, 2003, vol. 53, pp. 173–187.

[12] M.A. Heller, E. Gentez, *Psychology of Touch and Blindness*, Psychology Press, New York 2014, pp. 132–149.

[13] A. Sfard, *Balancing the unbalanceable: The NCTM Standards in Light of Theories of Learning Mathematics*, [in:] *A research companion to principles and standards for school mathematics* eds. J. Kilpatrick, W.G. Martin, D. Schifter, National Council for Teachers of Mathematics, Reston, Virginia 2003, pp. 353–392.

[14] B. Röder, F. Rösler, *Visual input does not facilitate the scanning of spatial images,* "Journal of Mental Imagery", 1998, vol. 22(3–4), pp. 165–182.

Studies on the use of analogies in a group of blind pupils, as well as the creation of spatial representations, were conducted by Marcus Knauff and Elisabeth May[15]. They have conducted three sets of experiments, in which blind and seeing persons (blindfolded) performed the same tasks. These applied to the determination of eye-space relations that are easy to solve both with eyesight as well as spatially; visual relations that can be solved easily visually, and difficult problems, in which the solutions are difficult to foresee both visually as well as spatially. As it turned out, the blind, irrespective of the task type, solved them more slowly and less precisely. The authors also believe that if inference on spatial relations is also based on verbal data, then the reasoning process itself proceeds more effectively and is more correct.

Analogical reasoning plays an enormous role in the life of the blind. It permits the cognition of phenomena that are not available to the blind by touch or hearing. As the seeing utilise verbal material and expressions describing relations between all senses – the blind must, as best as they can, get to know these expressions for themselves. Analogy lets them to just this. Thinking by analogies permits the cognition of specific relations between objects, concepts, phenomena, expands the vocabulary, increases the volume of understood concepts, thanks to which the emerging vision of reality is much more fitting.

Methodological assumptions

The purpose of the article is the determination, how reasoning by geometric analogies proceeds in three age groups of blind and seeing pupils (ages 10, 12 and 14). Based on literature concerning special education and psychology as well as own experiences, the following research questions were posed, which were later verified through appropriately selected measurement tools:

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[15] M. Knauff, E. May, *Mental Imagery, Reasoning, and Blindness,* "The Quarterly Journal of Experimetal Psychology", 2006, vol. 59(1), pp. 161–177.

1) What are the characteristics of reasoning by geometric analogies in blind pupils in the 3rd and 5th grades of primary school and in the 1st grade of lower secondary school?
2) What are the characteristics of reasoning by geometric analogies in blind pupils as compared to seeing pupils in the 3rd and 5th grades of primary school and in the 1st grade of lower secondary school?

Research on analogies based on geometric material was conducted, among others, by Bogdan Pietrulewicz[16]. He was able to show that in terms of tactile perception of the blind and seeing in 3rd and 5th grades of primary school and in first grade of lower secondary school, with reference to analogies based on image and geometric material, there are no statistically significant differences. In terms of tactile and visual perception of seeing pupils, in turn, there are statistically significant differences irrespective of the age of those studied (10, 12 and 14). In addition, work by B. Pietrulewicz[17] has shown that there are no statistically significant differences in terms of tactile perception of the blind and visual perception of the seeing. It is thus assumed that the blind can just as aptly as the seeing describe relations of analogies based on geometric material.

Characteristics of the studied group

The study spanned 126 pupils[18]. The basic group was composed of 63 blind pupils aged ten (3rd grade of primary school), 12 (5th grade

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[16] B. Pietrulewicz, *Rozwój rozumowanie przez analogię u dzieci niewidomych,* Wydawnictwo PAN, Komitet Nauk Psychologicznych, Zakład Narodowy Ossolińskich, Wrocław, Kraków, Gdańsk, Łódź 1983, pp. 82–95.

[17] B. Pietrulewicz, *Rozwój rozumowanie przez analogię u dzieci niewidomych,* Wydawnictwo PAN, Komitet Nauk Psychologicznych, Zakład Narodowy Ossolińskich, Wrocław, Kraków, Gdańsk, Łódź 1983, pp. 82–95.

[18] The study was conducted in the academic year 2007/2008 and are a part of studies executed ahead of a doctoral dissertation. Since that time, no studies concerning geometric analogies were undertaken in Poland.

of primary school) and 14 (1st grade of lower secondary school). They were subdivided into three age categories of 21 pupils. Blind pupils originally came from all of Poland and learned at seven special-purpose school facilities, hence their choice for the group was purposeful. The blind pupils were an equal group in terms of gender – there were 25 boys and 38 girls. Among the 21 studied third-graders – nine (42.9%) were at the boarding school for four years, meaning, since beginning learning in class „0", two persons (9.5%) remained there for three years, seven persons (33.3%) – two years, two persons (9.5%) – one year. In 5th grade, four persons (19%) resided at the facility since age six, seven (33.3%) – since 1st grade, one (4.8%) – since 2nd grade, five (23.8%) – since 3rd grade, 2 (9.5%) – since 4th grade, two persons (9,5%) had never stayed at a boarding school. Among pupils of lower secondary schools, eight (38.1%) remained at the boarding school for seven years, meaning since 1st grade, six (28.6%) – for six years, 3 (14.2%) – for five years, one (4.8%) – for four years, 1 (4.8%) – for a year, two lived outside of the facility.

The comparison group was composed of 63 seeing pupils subdivided into the same age groups of the same size. The choice for the comparative group was purposefully random, as it was selected in terms of numbers, genders, ages and places of residence as compared to the blind pupils.

When choosing persons for the group, the formerly selected criteria were adhered to: the presence or lack of eyesight damage (in case of pupils with eyesight damage, the studied group was made up of persons who were born blind or those who had lost their eyesight before the age of five); age (10; 12 14 years); determined correct intellectual development; school type (special education facility for blind pupils; public school for seeing pupils); no additional illnesses.

The present article presents results of trials spanning analogies based on image and geometric material. In order to test reasoning through geometric analogies, utilised were twelve *series B matrices* from the *Progressive Matrices* of John C. Raven. Only series B was selected because it is based on analogies between pairs of figures, which is congruent with the set research objectives. The seeing pupils were to solve the image-based version in its original form, with the blind receiving the matrices transformed into tactile versions.

The series B of 12 matrices by John C. Raven was constructed based on analogies between two pairs of figures. The task of the pupil is to discover the relation between the pair of figures, and the transposition of this relation on to the other pair, composed of one component that is shown, and the unknown to be chosen by analogy out of six indicated sections. The figures may be rotated about the horizontal or vertical axis. The shapes of the figures in tasks B_1, B_9, B_{10}, B_{11}, B_{12} are point-symmetric, and in tasks B_2, B_3, B_4, B_5, B_6, B_7, B_8 they are point-asymmetric. Point-symmetric figures do not change their position when rotated about their axis, and point-asymmetric figures do change their position.

Results

A statistical analysis of the data obtained thanks to the use of the *12 series B tables of Progressive Matrices by* J.C. Raven permitted the determination of the results in terms of geometric analogies achieved by both blind as well as seeing pupils aged ten, 12 and 14, learning on the same levels of education (see Table 1).

Table 1. Differences of means for the variables for geometric analogies of blind and seeing pupils

Variable	Pupil age	Blind pupils (N=63)		Seeing pupils (N=63)		Student's t-test results		
		M	SD	M	SD	t	df	p
Geometric analogies	10 years	4.10	1.37	6.00	2.53	–3.032	40	0.005
	12 years	7.76	2.30	8.48	1.81	–1.119	40	0.270
	14 years	9.33	1.24	10.00	1.55	–1.540	40	0.131

Legend: M – arithmetic mean, SD – standard deviation, df – degrees of freedom, p – significance, bold type – statistical significance < 0.05;

Source: Own analysis based on SPSS 24.0.

In terms of geometric analogies, the pupils were to solve 12 tasks. For each correct answer, a point was awarded. The maximum score was 12, the minimum – zero. The lowest mean value in geometric analogies was achieved by blind pupils attending 1st grade of lower secondary school (14 years), at M=9.33, which is 77.75% of the maximum achievable score (seeing pupils achieved a mean result of M=10). The results diverge by 1.23, and the range for the mean results is 8.10 to 10.56. The lowest score in the group of blind 14-year-olds is seven points, the top score – 11. The mean value of results achieved by blind pupils aged 10 was M=4.10, which is 34.16% of the maximum achievable result. This is the lowest score achieved among all the blind pupils. The results diverge by 1.37 points, and the range of mean results thus created is 2.73 to 5.47. In this group the minimum score was two points, and the maximum – seven.

The conducted analysis had disclosed the presence of one statistically significant difference between the studied groups of blind and seeing pupils (see Table 1). Based on Student's t-test, it was shown that in terms of reasoning through geometric analogies, there exists a statistically significant difference between blind and seeing pupils aged ten, learning in 3rd grade of primary school ($t=-3.032$ (40); $p<0.05$). Based on the mean arithmetic value, one could state that seeing pupils (M=6.00) achieve significantly higher results for the discussed analogy type.

The numerical data from table 1 permit the conclusion that there are no statistically significant differences between groups of blind and seeing pupils aged 12 (attending 5th grade of primary school) and 14 (1st grade of lower secondary school). One can only conclude, based on the arithmetic means, that both for the former (M=7.76 and M=9.33) and the latter age group (M=8.48 and M=10.00), seeing pupils achieve better scores in solving tasks based on geometric material.

The obtained empirical material also permits the conclusion that the highest distribution of scores among blind pupils is found in 5th grade of primary school (age 12), and among the seeing – in 3rd grade of primary school (age 10). This distribution reduces with age.

Based on arithmetic means, one may notice that the highest gain in results for the base group (blind pupils) as well as the control group (seeing pupils) is found between the 3rd and 5th grades of primary school.

Below is presented data concerning the intra-group difference determination using single-factor variance analysis (ANOVA) (see Table 2). Result F of the variance analysis for the variable of geometric analogies of blind pupils aged 10 (3rd grade of primary school), 12 (5th grade of primary school) and 14 (1st grade of lower secondary school) indicates the presence of statistically significant differences for the analysed variable, indicating intra-group variability.

Table 2. Results of the single-factor variance analysis (ANOVA) for the variable of age for geometric analogies in blind pupils

Variable	Group	M	Variance analysis (ANOVA) results			
			F	Groups	Difference of means	p
Geometric analogies	aged 10	4.10	F=52.235 df_1=2 df_2=60	1 and 2	–3.667	0.000
	aged 12	7.76		1 and 3	–5.238	0.000
	aged 14	9.33		2 and 3	–1.571	0.029

Legend: M – arithmetic mean, F – variance analysis factor, p – significance, bold type – statistical significance <0.05; df_1, df_2 – degrees of freedom.

Source: Own analysis based on SPSS 24.0

The analysis of the numbers had revealed, based on F factor value for the single-factor variance analysis (ANOVA) that reasoning through geometric analogies is differentiated by the age of the analysed persons ($F(2;60)=52.235$; $p<0,05$; see Table 2). For the purpose of determination of statistically significant differences in terms of reasoning by geometric analogies between blind pupils aged 10, 12 and 14, an analysis of the results was conducted using Dunnett's Test, as the variances were not uniform. It must be noted that pupils aged 14 (1st grade of lower secondary school) are characterised by a significantly higher mean result ($p<0.005$) compared to pupils aged 12 (5th grade of primary school; $p<0.005$) and pupils aged 10 (3rd grade of primary school; $p<0.05$). One may additionally conclude, based on arithmetic means, that the higher the age difference between pupils, the discrepancies among the groups of blind people are higher.

The analysis of the results also shows that for blind people in the three studied age groups, the following turned out to be the simplest tasks: 1 (M=18.33), 2 (M=17), 3 (M=17), 4 (M=16), 5 (M= 14.33), 6 (M=13), 7 (M=10.66) and 9 (M=11.33). Difficult were the following: 8 (M=9.66), 10 (M=10), 11 (M=6.66), 12 (M=4.31). For seeing pupils, the tasks solved flawlessly are: 1 (M = 20.66), 2 (M=20.66), 3 (M=20.33), 4 (M=17.66), 5 (M=16.33), 6 (M=15), 7 (M=13), 9 (M=11.33) and 10 (M=11). The tasks that proved most difficult are 8 (M=10), 11 (M=7.66) and 12 (M=7.66). There was no task, in which blind pupils would score better than seeing pupils.

Below the individual tasks are characterised according to their numbers.

Task 1 (concerns a symmetrical figure) was the easiest of the 12 presented tasks, and did not pose too much trouble both for blind as well as for seeing pupils. Solely blind third-graders achieved a score of just 13 (61.9%). Almost all of the rest answered correctly.

Task 2 was constructed according to a similar principle as the former, but was somewhat more complicated (point-asymmetric figure). The largest difference are visible between 3rd grade (11 points – 52.38%) and 5th grade (19 points – 90.47%) in the blind group. This difference is eight points. In the group of seeing pupils, this difference is just one point. Pupils of both groups in 1st grade of lower secondary school scored 21 (100%). The typical error made by the pupils is indicating a figure close to the correct one, but differing in size and proportions of the individual components (answers two and three).

Task 3 concerned a point-asymmetric figure. The basis for the correct solution is uncovering the relation of opposition between the pairs of figures. The spread of results is similar to task 2. The highest difference was found between 3rd grade (11 points – 52.38%) and 5th grade (19 points – 90.47%) in the group of blind pupils. There were no such differences in the group of seeing pupils. The highest volume of wrong answers given by blind people applied to figure 5, because they did not take into account the 180˚ rotation. They also indicated figures 4 (figure of varied properties) and 2 (smaller figure).

Task 4 concerned a point-asymmetric figure. Its solution is dependent on the ability to perform the synthesis of a circle. Differences between groups are becoming more visible. Blind third-graders achieved a result of 10 (47.61%) and seeing pupils – 14 (66.66%). The difference amounted to 8 points between 3rd and 5th grade of blind pupils, five for the seeing group. Between 3rd grade of the blind and seeing pupils the difference was four points, and the difference between blind and seeing fifth-graders – one point. Both pupils with damaged eyesight as well as seeing pupils of the 1st grade of lower secondary school achieved scores of 20 (95.23%). The most common wrong answers were indications of figures 4 and 6 as correct. In these figures rotation is not taken into account, they are repetitions of original figures. Figure 3, a smaller-sized figure, was also chosen.

Task 5 s also a point-symmetric figure with varying properties. The analysed person, in order to solve this task, would need to rotate this figure and take into account the altered property of surface linearity. This is a task that is quite difficult for younger pupils. In all age categories, seeing pupils achieved higher results as compared to pupils with damaged eyesight. Both in the base as well as in the control groups, the result improvement progresses most between 3rd and 5th grade of primary school. The most common mistakes made by younger pupils in both studied groups was indicating tables four and five. These are figures that are not rotated by 180°. Figure 3 is the same as the main figure from the bottom left corner. Figure 5 reflects the upper right figure, to which the pupil is supposed to find an analogy.

Task 6 is point-asymmetric, and entails rotation of the figure by 180°. The biggest improvement of results in the base and control groups was found between 3rd and 5th grade of primary school. The difference in scores in the blind group was eight points, ten points in the seeing group – 10 points. Differences between results achieved by 5th grade of primary school and 1st grade of lower secondary school are minimum and amount to two points and one point, respectively. Characteristic errors made by pupils in both groups is indicating the following answers: 2, 4, 5, 6. Figure 2 corresponds to the main figure in the upper left corner, figure 4 is equivalent to the main figure from the upper right part of the page. Responses indicating figures 5 and 6 apply to rotation by 180°, not by 90°.

Task 7 also covers a point-asymmetric figure. The difficulty level for this task is high, requiring the pupil to execute a rotation by 180° and to consider the surface linearity, which is not simple for blind pupils in lower grades. The highest improvement of scores for this task is again found in the time between 3rd and 5th grade of primary school. The difference in results between 3rd and 5th grade in the group of blind pupils amounted to seven points, and in the group of seeing pupils – eight points. The difference between 5th grade of primary school and 1st grade of lower secondary school in the group of blind pupils is three points, and one point in the group of seeing pupils. In this task, for all age groups, seeing pupils achieved better results. The answers selected most frequently were 2, 3, 4 and 6. Figure 2 repeats the main figure from the upper right part of the page. Figure 3 is rotated horizontally and vertically by 180°, but has no characteristics of linearity. Figure 4 is rotated by 180° in the horizontal axis. Figure 6 is rotated by 180° in the horizontal and vertical axes, but has characteristics of linearity.

Task 8 concerns a point-asymmetric figure. Its solution is dependent on the ability to discern a figure from its background. The highest gain in results is found between 3rd and 5th grade of primary school. In the blind group, the difference in scores between pupils of these grades was 5 points, among seeing pupils – 8 points. Differences between pupils from 5th grade of primary school and 1st grade of lower secondary school are minimal. In this task, seeing pupils again achieved better results than their blind counterparts.

Errors concerned the following figures: 1, 3, 5, 6. Figure 1 is a repetition of the main figure in the top part of the left page. Figure 3 refers to main figures placed on the right in the bottom and upper parts. Pupils who chose response no. 5 as the unknown, indicated the figure but omitted changes in its properties. Figure 6 may be similar to the correct one, but is not identical.

Task 9 refers to a point-symmetric figure and requires rotation of figures. In this task, just like in the former tasks, the highest spread of results both for blind pupils as well as for seeing pupils is found between 3rd and 5th grades of primary school. The difference in the achieved results for the blind pupil group was 5 points, and in the group of seeing pupils – 7 points. Differences between 5th grade and grade one of lower secondary school are minimal. In this task, blind people from 3rd grade got one point more than seeing pupils. However, the score of pupils from the basic and comparison group from the 1st grade of lower secondary school is 15 (71.42%) each. Wrong solutions are the following figures: 1, 2, 5, 6. Figure 1 is a repetition of the main figure on the upper right side of the page. Figure 2 is also a repetition of the main figure, to which analogies must be made, but it does not consider changes of properties due to added elements. Figure 5 is a repetition of the main figure from the lower left part of the page. Figure 6 considers properties of the main figure, but not the one to which analogies must be found.

Task 10 is a point-symmetric task, and requires the introduction of an additional element. The highest gain of results is found for the period between 5th grade of primary school and 1st grade of lower secondary school, because for blind pupils the difference here is eight points, and for seeing pupils – 12 points. This task turned out to be very difficult for blind third-grade pupils, because they only scored four points. Among the groups of blind and seeing pupils, mistakes were similar. Most mistakes were made by blind 3rd grade pupils. They indicated all the possible answers, most frequently – 1 and 2, repetitions of the main figures. Figures 3, 4, 5 and 6 were chosen less frequently.

Tasks 11 and 12 are point-symmetric. Both pupils from the base group as well as from the control group achieved low scores. In

task 11, which entailed changing the location of the figure, blind and seeing third-graders only gave two correct answers. In task 12, none of the blind pupils chose a correct answer. Neither in task 11, nor in task 12 did blind pupils achieve better results than those of seeing pupils. In terms of task 11, pupils would indicate all of the possible wrong answers, with figures 2, 3 and 6 being most frequent. Figure 2 was a repetition of the main figure from the left part of the page; figure three is a rotation of the figure, to which an analogy was to be constructed. Responses indicating figure 6 referred to a repetition of the main figure. Task 12 was the most difficult. Its basis was subtraction of figures in the centre. The most common errors were answers two and three, which were repetitions of the main figures. They appear most frequently, because pupils, not finding analogies, called on the figures to which they were supposed to find references. Responses 1 and 4 were decidedly less frequent.

Summary and conclusions

To summarise, results obtained during studies on reasoning based on geometric analogies permit the following conclusions:

1. On the basis of statistical analyses it could be shown that in terms of reasoning by geometric analogies, there exists a statistically significant difference between blind and seeing pupils in the 3rd grade of primary school (age 10), in favour of seeing pupils. Blind fifth-graders (age 12) and pupils of the 1st grade of lower secondary school (age 14) also achieved lower scores than their seeing peers, yet these are not statistically significant differences. Such results may be explained by slower development of brain operations such as comparisons, in blind pupils. With age, the distribution between the base and comparison group drops decidedly, as older-aged pupils,

thanks to systematic tactile exercises, perfect their abilities to differentiate, classify and rotate objects. Similar conclusions were drawn, based on their research, by Bogdan Pietrulewicz[19] and Krzysztof Klimasiński[20].

2. In all 12 trials of geometric analogies, it was determined that blind pupils achieved weaker scores than their seeing peers did. This could stem from the properties of the material they worked with. Seeing pupils solved original tasks, and blind pupils used tactile versions, making noticing differences between the figures, as noted earlier, more difficult.
3. Point-symmetric trials turned out to be the most difficult, as they required changing the location of the figure and subtraction of figures from the inside. The most common errors made both by blind as well as by seeing pupils were the following: repetitions of main figures, wrong rotation, choosing similar figures that differed in size and proportion of the individual components, failing to take account linearity. Errors had similar properties in both groups, but blind people made many more of them.

Analogical reasoning is one of the basic modes of expansion and enrichment of knowledge. It is the more perfect, the more blind pupils have modes to utilise it in practice. For this purpose, teachers should care for the preparation of various tasks of analytical, synthetic and analytical-synthetic character.

Teachers should also keep in mind that during classes not only in exact sciences, pupils should be able to make comparisons, abstractions, to rotate and change the location of items. In order to improve the quality of geometric analogies, as many sensory exercises as possible are needed; these should be based on the senses of touch and hearing as well as spatial orientation.

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[19] B. Pietrulewicz, *Rozwój rozumowanie przez analogię u dzieci niewidomych,* Wydawnictwo PAN, Komitet Nauk Psychologicznych, Zakład Narodowy Ossolińskich, Wrocław, Kraków, Gdańsk, Łódź 1983, pp. 82–95.

[20] K. Klimasiński, *Rola wyobrażeń przestrzennych w rozwoju myślenia dzieci niewidomych,* Wydawnictwo PAN, Zakład Narodowy Ossolińskich, Wrocław–Warszawa–Kraków–Gdańsk 1977, pp. 47–89.

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