

SERGIUSZ HERMAN

## A COMPARATIVE ANALYSIS OF THE PREDICTABILITY OF SELECTED METHODS FOR PREDICTING BUSINESS FAILURE\*

### I. INTRODUCTION

The issue of forecasting business failure worldwide has been known since the beginning of the twentieth century. The rapid increase in this interest goes back to the 1920s and 1930s and the economic crisis prevailing at that time. In the United States, the main focus of the research was the financial statements of enterprises and the use of information drawn from them for the purpose of forecasting potential insolvencies of companies. The first ones to do so in 1930 were Smith and Winakor.<sup>1</sup> They studied changes in the value of 24 financial ratios that described the situation of 29 companies threatened with insolvency proceedings. On the basis of the results obtained they selected a group of 8 ratios of high prognostic value. Two years later, a similar analysis was carried out by Fitzpatrick.<sup>2</sup> This time the values of the financial ratios were compared for two groups: 19 companies that were going concerns and 19 insolvent ones. Based on the study, two ratios (the financial result/equity, and equity/external capital) were identified, and their values made it possible to predict the insolvency of enterprises in the most efficient way.

Merwin continued similar studies and elaborated on them,<sup>3</sup> selecting, on the basis of a comparative analysis, three financial ratios: the working capital/asset value, equity/external capital and current assets/current liabilities. Their average values for healthy companies and for those declared insolvent were clearly different as early as six years before the crisis situation arose. This research direction was continued by Beaver.<sup>4</sup> He analysed the predictive capacity of 30 financial ratios and suggested that

---

\* Translation of the paper into English has been financed by the Minister of Science and Higher Education as part of agreement no. 848/P-DUN/2018. Translated by Iwona Grenda.

<sup>1</sup> R.F. Smith, A.H. Winakor, A test analysis of unsuccessful industrial companies, *Bureau of Business Research* 1930, no. 31.

<sup>2</sup> P.J. Fitzpatrick, A comparison of ratios of successful industrial enterprises with those of failed companies, *The Certified Public Accountant* 1932: 598–605.

<sup>3</sup> C. Merwin, *Financing small corporations in five manufacturing industries, 1926–1936*, National Bureau of Economic Research 1942.

<sup>4</sup> W. Beaver, Financial ratios as predictors of failure, *Journal of Accounting Research* 5, 1966: 71–111.

the simultaneous use of multiple ratios should guarantee a higher accuracy than when forecasting based on variables analysed in isolation.

A milestone in the development of the research into forecasting insolvencies worldwide was the publication of Altman in 1968<sup>5</sup> who was the first to propose the use of a multidimensional discriminant analysis, thus putting into practice Beaver's idea (concerning the simultaneous inclusion of multiple variables in forecasting.) Altman developed a tool which he called a Z-score, which was intended for forecasting insolvencies of industrial companies through the use of five financial indicators. Altman's pioneering publication initiated a new, multidimensional direction of research. In the following years Martin<sup>6</sup> proposed the use of logistic regression models for predicting insolvencies, while Żmijewski<sup>7</sup> proposed the use of probite models.

In the literature all methods of forecasting the insolvency of enterprises presented so far are counted as statistical methods. The main weakness of these methods is the numerous assumptions which they entail, such as normal distribution or the independence of predictive variables associated with them. Over years, artificial intelligence techniques, freed from these restrictive assumptions, became increasingly popular. Among them were those first used by Odom and Sharda,<sup>8</sup> such as neural networks, genetic algorithms,<sup>9</sup> support vector machines,<sup>10</sup> or decision trees.<sup>11</sup>

The frequency of the usage of particular methods of forecasting insolvencies in empirical research changed over time. It was researched by, among others, Aziz and Dar.<sup>12</sup> On the basis of their study, it can be concluded that the most popular were the statistical methods (used in the research reported in 64% of publications analysed), followed, respectively, by soft computing techniques (25%) and theoretical models (11%). Aziz and Dar also analysed in detail the frequency in which these methods were used. Graph 1 shows those that were most frequently used.

---

<sup>5</sup> E.I. Altman, Financial ratios, discriminant analysis and the prediction of corporate bankruptcy, *The Journal of Finance* 23(4), 1968: 589–609.

<sup>6</sup> D. Martin, Early warning of bank failures: a logit regression approach, *Journal of Banking and Finance* 1, 1977: 249–276.

<sup>7</sup> M. Żmijewski, Methodological issues related to the estimation of financial distress prediction models, *Journal of Accounting Research* 22, 1984.

<sup>8</sup> M.D. Odom, R. Sharda, A neural network model for bankruptcy prediction, *Proceedings of IEEE International Conference on Neural Networks* 1990: 151–173.

<sup>9</sup> K.S. Shin, Y.J. Lee, A genetic algorithm application in bankruptcy prediction modeling, *Expert Systems with Applications* 23(3), 2002: 321–328.

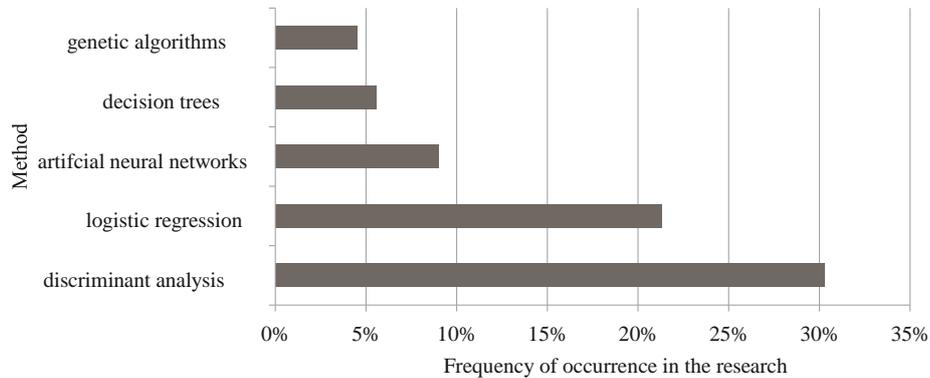
<sup>10</sup> K. Shin, T.S. Lee, H. Kim, An application of support vector machines in bankruptcy prediction model, *Expert Systems with Applications* 28, 2005: 127–135.

<sup>11</sup> A. Gepp, K. Kumar, Predicting financial distress: a comparison of survival analysis and decision tree techniques, *Procedia Computer Science* 54, 2015: 396–404.

<sup>12</sup> M.A. Aziz, H.A. Dar, Predicting corporate bankruptcy—where we stand?, *Corporate Governance Journal* 6(1), 2006: 18–33.

**Graph 1**

The frequency of the application of particular methods in studies forecasting insolvencies



Source: author's own studies based on: Aziz, Dar, op. cit.

The results obtained by Aziz and Dan were heavily dependent on the period from which the articles and other publications which they studied came from—all were made available before 2003.

A similar study was conducted by Kirkos.<sup>13</sup> This time the subject of the analysis was 118 scientific articles published in the period 2009–2011. As the research results showed, in as many as 55% of the publications researched the methods used were based on artificial intelligence, a consequence of the development of information technologies. Among the most frequently used methods were support vector machines and artificial neural networks.

There have been numerous repeated attempts to compare the prognostic capacity of different business failure predicative models constructed using various methods of forecasting the insolvency of enterprises. Among comparative studies reported in foreign-language literature that involved different models forecasting insolvencies, are works by Ravi Kumar and Ravi<sup>14</sup> who conducted a comprehensive review of 128 reports. Polish literature contains few publications intended to compare the predictive capacity of different methods of forecasting insolvencies.

The main objective of the empirical study presented below is a comprehensive comparison of the predictive capacity of nine selected methods of forecasting insolvencies of enterprises in Poland. For this purpose, the forecasting ability of a number of models constructed with the use of these methods has been evaluated. In this study, predicative ability was measured using the global coefficient of accurate classification (which is generally

<sup>13</sup> E. Kirkos, Assessing methodologies for intelligent bankruptcy prediction, *Artificial Intelligence Review* 43(1), 2015: 83–123.

<sup>14</sup> P. Ravi Kumar, V. Ravi, Bankruptcy prediction in banks and firms via statistical and intelligent techniques—a review, *European Journal of Operational Research* 180, 2007: 1–28.

referred to as accuracy) and the global coefficient of misclassification (hereinafter referred to as a prediction error). Predictive capacity was compared using many research samples, taking into account different statistical methods of selecting variables for models, different sample sizes, as well as different forecast timescales. This has been the first survey of such wide scope conducted in Poland so far.

The article is divided into three parts. The first one is an overview of the Polish empirical studies, whose authors used different methods to forecast the insolvency of companies. In the next part the sample and the research methodology are described, while the last part presents the results of the empirical survey, closing the deliberations with some conclusions.

## II. THE LITERATURE REVIEW

As mentioned in the introduction, there are few scientific publications in the Polish literature in which an attempt was made to compare the predictive capacity of models constructed by the use of various methods of forecasting the insolvency of enterprises. The first author who decided to use more than two statistical methods for this purpose was Hołda.<sup>15</sup> The sample he used consisted of 374 companies, representing three sectors of the economy: manufacturing, construction and trade and commercial services. This sample was divided into a learning and a test group, with a ratio of 70:30. In his study, Hołda used four methods of forecasting company insolvency, namely a linear discriminant analysis, a logistic regression, neural networks and decision trees, and three different methods for selecting variables. The results obtained in the study led to the conclusion that there is no single method or technique of building insolvency prediction models which would have a systematic advantage over the others.

Korol was another author who decided to use various prognostic methods in his research. To this end, he analysed a group of 185 companies in the services and manufacturing sectors, of which 53 companies were a learning sample. The author of the study used two test samples of 54 and 132 companies, respectively. In the first empirical analysis, Korol used the following prognostic methods:<sup>16</sup>

- a recurrent artificial neural network (SSN REC),
- a one-way multilayer artificial neural network (SSN MLP),
- an artificial neural network based on genetic algorithms (SSN GA),
- an artificial neural network with radial base functions (SSN RBF),
- a self-organising map (SOM),

---

<sup>15</sup> A. Hołda, *Zasada kontynuacji działalności i prognozowanie upadłości w polskich realiach gospodarczych*, Cracow: AE w Krakowie, 2006.

<sup>16</sup> T. Korol, Prognozowanie upadłości firm przy wykorzystaniu miękkich technik obliczeniowych, *Finansowy Kwartalnik Internetowy e-Finanse* 2010, no. 1.

- a support vector machine (SVM),
- a fuzzy logic model (FL).

The study was an attempt to verify the effectiveness of a wide range of methods included in the group of soft computing techniques in forecasting the insolvency of companies in Poland. The results of the analysis clearly showed that in the case of using the proportion of enterprises that was close to the real conditions of the economy, the model of fuzzy logic has a higher predictive capacity than other models of artificial intelligence.

In the other study, an extended version of the analysis presented, Korol<sup>17</sup> added to the group of methods studied previously the linear discriminant method, logistic regression methods and classifications trees, and the random forest. As before, in the case of a larger test sample, fuzzy logic models predicted most accurately the failure of the companies surveyed. In the case of a smaller test sample, the results obtained depended on the forecast timescale.

The authors of another publication worthy of mentioning here as part of the literature review, were Pocięcha, Pawełek, Baryła and Augustyn.<sup>18</sup> These authors had at their disposal research samples of between 118 and 246 companies. In the study, they were divided into learning and test samples in proportions of 7:3 and 6:4. The empirical analysis was carried out in three variants:

- a) Variant I—forecasting business failure one year in advance (grouping together companies which were declared insolvent in years 2007–2010);
- b) Variant II—forecasting business failure 2 years in advance (grouping together companies which were declared insolvent in the period 2007–2010);
- c) Variant III—forecasting business failure 2 years in advance (taking into account only those enterprises which were declared insolvent in 2009).

Two methods of research sample selection were used in the study: the evaporation method and the method of independent sampling. The authors compared the predictive capacity of models constructed with the use of 4 methods of forecasting business failure: a linear discriminant analysis, a logistic regression, the CART decision tree and a neural network. Taking into account all the above assumptions of the research conducted, a total of 12 rankings of the comparable models were obtained. The one that most often appeared at the top in the ranking was the neural network-based model. In as many as 11 cases it was at the top of the rankings created. Only in one case did the model constructed by the method of classification trees give more accurate forecasts. The worst forecasting capabilities of the first two survey options were found in statistical models. In the third variant of the test, the method with the lowest predictive abilities was the classification tree.

The last author who decided to apply various research methods in his research related to forecasting companies failure was Gaska. In his first

---

<sup>17</sup> T. Korol, *Systemy ostrzegania przedsiębiorstw przed ryzykiem upadłości*, Warsaw: Oficyna, 2010.

<sup>18</sup> J. Pocięcha et al., *Statystyczne metody prognozowania bankructwa w zmieniającej się koniunkturze gospodarczej*, Cracow: Fundacja UE w Krakowie, 2014.

empirical study<sup>19</sup> he used a research sample consisting of only 36 public limited companies. Owing to the small size of the surveyed group the following methods of estimating the prediction error of the constructed models were used in the survey:

- the method of resubstitution,
- the method of 5-fold cross-validation,
- the method of  $n$ -fold cross-validation,
- the bootstrap method 0.632.

In his analysis, Gaška compared the predictive capacity of models constructed according to the load-bearing vector method with the results obtained for logistic regression methods and a one-dimensional reference model.<sup>20</sup> The lowest prediction errors (and thus the highest prediction capability) were obtained for the support vector machine with the Gaussian kernel function.

In his next study Gaška<sup>21</sup> decided to extend the scope of the study previously presented. This time, the research sample consisted of a group of 94 public limited liability companies, while the range of comparable methods was as follows:

- logistic regression,
- random forest,
- $k$ -nearest neighbours method
- support vector machine with Gaussian kernel function,
- support vector machine with a polynomial kernel function,
- fuzzy margin classifiers method with the Gaussian kernel function. The prediction error was estimated using a 5-fold cross-validation. The highest predictive capacity was observed in the method of random forests and the method of fuzzy classifiers of the maximum margin.

The common feature of the first three studies presented in this part of the article is that their authors used only a simple method of division in order to estimate the predictive capacity of models constructed with the use of the prognostic methods studied.

The results of numerous empirical studies conducted around the world clearly indicate that using this method leads to a very high variability of results.<sup>22</sup> The literature emphasises that a simple method of division should only be used if there is a sufficiently broad set of data allowing a sufficient number of independent sets to be distinguished: the training one and the test

---

<sup>19</sup> D. Gaška, Zastosowanie metody SVM do oceny ryzyka bankructwa i prognozowania upadłości przedsiębiorstw, *Śląski Przegląd Statystyczny* 2013, no. 11: 289–310.

<sup>20</sup> Selection on the basis of a learning sample of the most discriminating traits, setting a threshold value for it and classifying objects from the test sample on its basis.

<sup>21</sup> D. Gaška, Prognozowanie bankructwa za pomocą klasyfikatorów rozmytych realizujących ideę maksymalnego marginesu, *Śląski Przegląd Statystyczny* 13(19), 2015: 71–88.

<sup>22</sup> For example U.M. Braga-Neto, E.R. Dougherty, Is cross-validation valid for small-sample microarray classification?, *Bioinformatics* 20(3), 2004; J. Kim, Estimating classification error rate: repeated cross-validation, repeated hold-out and bootstrap, *Computational Statistics and Data Analysis* 53(11), 2009.

one.<sup>23</sup> The only one who decided to use a different method of estimating the prediction error in his research was Gaška. It should be noted, however, that his considerations were primarily aimed at proposing a new approach to predicting business failure based on the idea of the vector-bearing method. Other prediction methods used in the study were only a point of reference for the analyses carried out. The common denominator of all the empirical studies presented is the fact that the predictive capacity of models constructed with the use of different predictive methods was estimated only once, on the basis of a single test sample.

### III. TEST SAMPLE AND TEST METHODOLOGY

The empirical survey required an appropriate sample representing two distinct populations: businesses in poor condition (hereinafter referred to as: 'sick') and those of sound financial standing (hereinafter referred to as: 'healthy'). The decisive criterion for the classification of companies in the first group was the fact that the competent court had declared the insolvency of the company. In order to select the sample, the information contained in the online Monitor Sądowy i Gospodarczy [*Court and Commercial Gazette*] was used. In this way financial data were collected for 90 public limited liability companies (including: 30 public limited companies from the construction industry [PKD 41.10-43.99z]), 30 public limited companies from the industrial processing industry [PKD 10.11-33.20z] and 30 public limited companies from the wholesale and retail trade [PKD 46.11-47.99z]).

For each of these companies, a public limited liability company in good financial condition was selected. The matching criteria for each pair were: the sector of business, the main activity and the size of assets. The financial data of insolvent companies were derived from their financial statements for the year preceding that in which the first petition for insolvency was filed and for the two previous years. The financial statements were from 2000–2013. The financial statements of healthy companies that were examined were prepared for the same years. The sources of the data used were databases of Notoria Serwis and Bisnode Dun & Bradstreet and the Monitor Polski B [*Official Journal of the Republic of Poland*].

In the empirical research, 19 financial ratios characterising profitability, liquidity, capital and asset structure and the efficiency of enterprises were calculated and used (Table 1). Their selection was made on the basis of a literature review—these are ratios that appear most frequently in business failure prediction models. The choice of ratios was also determined by the availability of the data disclosed in companies' financial statements.

---

<sup>23</sup> For example B.D. Ripley, *Pattern recognition and neural networks*, Cambridge: CUP, 1996.

**Table 1**

List of financial ratios used in the survey

<b>Ratio</b>	<b>Formula</b>
ROA	net profit/ assets
ROE	net profit/ equity
ZB	gross profit/ assets
ZS	profit on sales/ sales revenue
MZ	gross profit/ sales revenue
MZ2	net profit/ sales revenue
MZO	operating profit/ sales revenue
KP	working capital/ balance sheet total
WBP	short-term assets/ short-term liabilities
WSP	(current assets—inventories)/ short-term liabilities
WPP	(working assets-inventories-receivables)/ short-term liabilities
ZO	total liabilities/ total assets
ZD	long-term liabilities/ total assets
KW	equity/ total assets
KWZ	equity/ total liabilities
RN	average value of receivables/ net sales revenues*365
RZ	average value of inventories/ net sales revenues*365
RZob	average value of liabilities/ net sales revenues*365
Rakt	average value of assets/ net sales revenues*365

Source: author's own studies.

In order to avoid any correlation of variables describing objects, it was assumed that each time before the research took place the variables that are strongly correlated with the others are removed (Spearman correlation coefficient higher than 0.90).<sup>24</sup> Additionally, to select the financial ratios with the highest discriminatory capacity, three popular statistical methods of variable selection were used in the study:

<sup>24</sup> If two variables are strongly correlated, the one for which the average of the absolute coefficients of correlation between this variable and the others is higher is removed from further analysis.

– selection of 5 variables, which had the highest absolute value of statistics  $t$  in the test comparing the average value of ratios in the groups studied—this method is further referred to as: *tstatistics*;

– selection of variables based on the strength of the dependence between individual financial indicators and a variable  $y$  describing the membership of an object in a specific group. The variables for which the Spearman correlation coefficient is statistically significant at the level of  $\alpha = 0.05$  were used; this method is further referred to as: *correlations*;

– a stepwise selection of variables in advance. It was assumed that in subsequent steps the variable which causes the greatest decrease in Wilk's Lambda statistic is included in the model.<sup>25</sup> Since the distribution of these statistics is very complex, the significance level of 0.1 for the value of  $F$  (being a good approximation of Wilks lambda) was assumed as a condition for the introduction of the variable. This method is further referred to as: *stepwise*.

As has been mentioned in the introduction, the purpose of the empirical study is to compare the predictive capacity of different methods of forecasting insolvency of companies. The predictive ability can be measured by a predictive error. The results of earlier research conducted by the author indicate that in the case of forecasting the insolvencies of public limited liability companies in Poland, the prediction error estimators obtained using the bootstrap method show the most desirable properties.<sup>26</sup> Consequently, the prediction error estimate  $+0.632$  proposed by Efron and Tibshirani in 1997 was used.<sup>27</sup>

The following nine methods of forecasting insolvencies were used for the comparative analysis of company failure:

- a linear discriminant function,
- a square discriminant function,
- logistic regression,
- naive Bayes classifier,
- the  $k$ -nearest neighbours method,
- classification trees,
- random forest,
- neural networks,
- support vector machine.

Their detailed description can be found in works by Koronacki and Ćwik or publications by Krzyśko, Wołyński, Górecki and Skorzybut, among others.<sup>28</sup>

<sup>25</sup> Lambda Wilks statistics determine the discriminatory power of the model, it may range from zero (excellent discriminatory power) to zero (total absence of discriminatory power).

<sup>26</sup> S. Herman, Analiza porównawcza wybranych metod szacowania błędu predykcji klasyfikatora, *Przegląd Statystyczny* 63(4), 2016.

<sup>27</sup> B. Efron, R. Tibshirani, Improvements on cross-validation: the .632 bootstrap method, *Journal of the American Statistical Association* 92(438), 1997.

<sup>28</sup> J. Koronacki, J. Ćwik, *Statystyczne systemy uczące się*, Warsaw: Wyd. Naukowo-Techniczne, 2005; M. Krzyśko et al., *Systemy uczące się: rozpoznawanie wzorców, analiza skupień i redukcja wymiarowości*, Warsaw: Wyd. Naukowo-Techniczne, 2008.

Most of the methods used required the values of the relevant parameters to be determined. Eventually, the values of parameters for which the method showed the lowest value of the estimated prediction error were used. And so, in the case of the  $k$ -nearest neighbours method, the number of neighbours in the range  $\langle 1;6 \rangle$  was considered, the final decision being that  $k = 5$ . For the classification trees, the minimum number of objects in the node (to make the division possible): 5, 10, 15, 20, and the minimum number of leaves: 1, 3, 5, 10 were considered in the study. Ultimately, classification trees were used, in which the minimum number of objects in the node was 5, while the minimum number of leaves equalled 1. The measure of the diversity of classes in the node was the Gini index. The analysis compared random forests built on the basis of 100, 200, 300, 400 and 500 trees, where the number of variables in the node could vary in the range  $\langle 1;5 \rangle$ . The results presented in the following section concern random forests built on the basis of 100 trees, where in each node only one variable was randomly drawn. In the case of neural networks, the form of a multilayer perceptron was used. The networks consisted of 3 layers (1 input, 1 hidden and 1 output). The number of input layer neurons corresponded to the number of financial ratios used to build individual networks. The number of hidden layer neurons was 3. The learning of the network was based on the BFGS algorithm. Different variants of the support vector machines were also considered in the analysis:

- with a linear kernel function;
- with the Gaussian kernel function, with the parameter value  $\delta \in \langle 0; 0.9 \rangle$ ;
- with a polynomial kernel function, with a degree of polynomial  $q \in \{1, 2, 3, 4, 5\}$ ;

The method of support vector machine with a linear kernel function exhibited the highest accuracy of forecasts and this method was used in the empirical analysis presented in the next part of the article. All calculations were made using the statistical environment *R*.

#### IV. RESULTS OF THE STUDY

The empirical study was conducted in three stages. The first one verified the accuracy of the classification and the ranking of the surveyed methods of predicting the insolvency of an enterprise depending on the method used to select forecasting variables. To this end, the following assumptions were made:

- a) the prediction error was estimated for 100 randomly drawn, stratified sub-samples of 100 companies,
- b) the forecast horizon was one year,
- c) three methods set out in the third part of the article were used: *tstatistics, correlations, stepwise*.

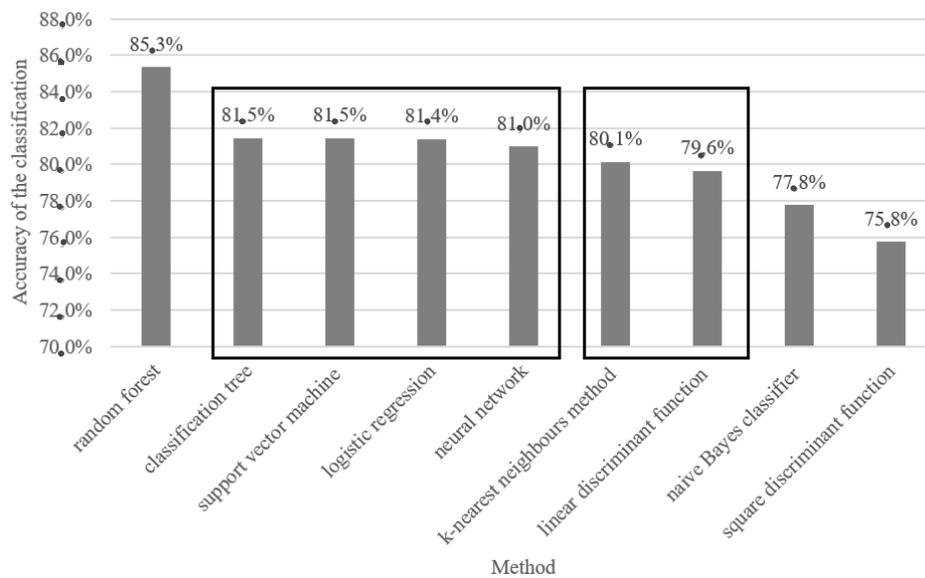
After estimating model prediction errors for 100 random sub-samples, using each of the analysed methods of forecasting insolvencies, the results were averaged. In order to verify whether the average accuracy of

classification is equal for the methods for predicting business failure analysed, a one-way analysis of variance was used. The Tukey test was used to determine which pairs of business failure forecasting methods had different classification accuracy levels.

Graphs 1–3 show the accuracy of classifications obtained using different, particular methods of predicting the insolvency of enterprises. After a one-way analysis of variance based on the values of  $F$  statistics, it was found that in all three cases (for the three methods of selecting predictive variables) there were grounds to reject the hypothesis that the level of accuracy of classification for all analysed forecasting methods was the same. The next step was to perform the Tukey test. As for the methods framed in the graphs, there was no reason to reject the hypothesis that their average classification accuracy level was the same.

**Graph 1**

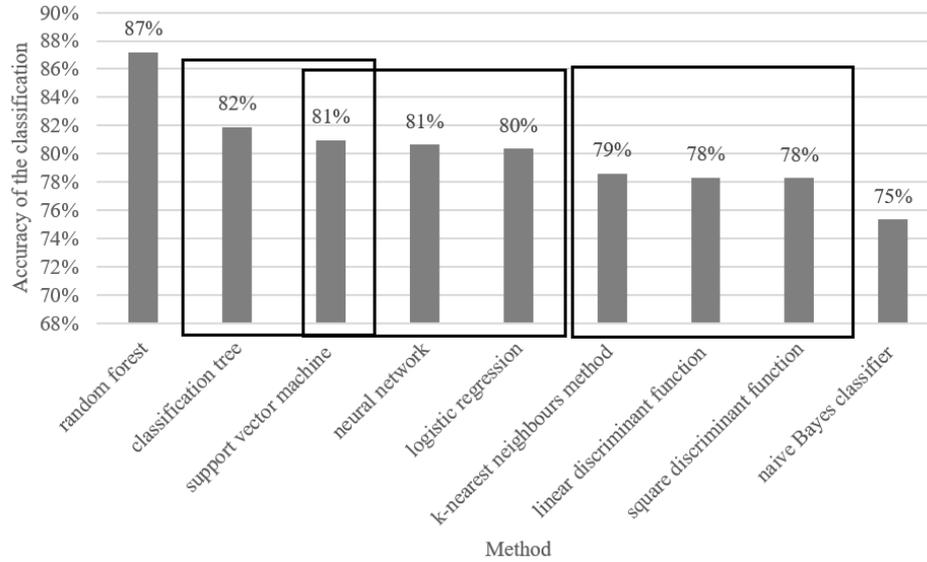
Accuracy of the classification of the methods of forecasting business failure—the variable selection method: *t* statistics



Source: author's own studies.

**Graph 2**

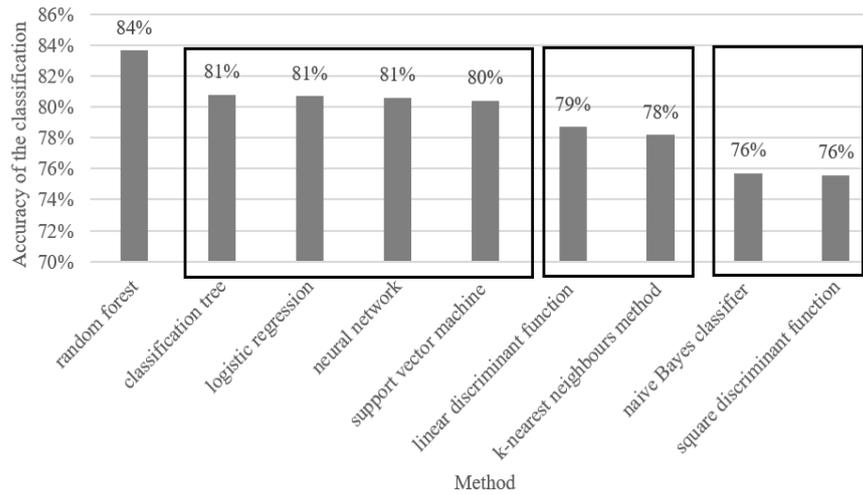
Accuracy of the classification of the methods of forecasting business failure—the variable selection method: *correlations*



Source: author's own studies.

**Graph 3**

Accuracy of the classification of the methods of forecasting business failure—the variable selection method: *stepwise*



Source: author's own studies.

A detailed analysis of the graphs leads to several interesting conclusions. The average level of the accuracy of classification for the analysed methods of selecting prognostic variables shows low variability. Regardless of the method of selecting variables, the random forest method is always characterised by the highest, and statistically significantly different from other methods, level of accuracy of classification. The least efficient methods of forecasting the companies' failure turned out to be the naive Bayes classifier and the square discriminant function. A change in the statistical method of selecting predictive variables for the model does not cause any significant changes in the accuracy of the classification of the methods studied, and consequently in their rankings.

At the second stage of the study, an attempt was made to verify the accuracy of the classification of models constructed using the prognostic methods tested, depending on the size of the study sample. For this purpose:

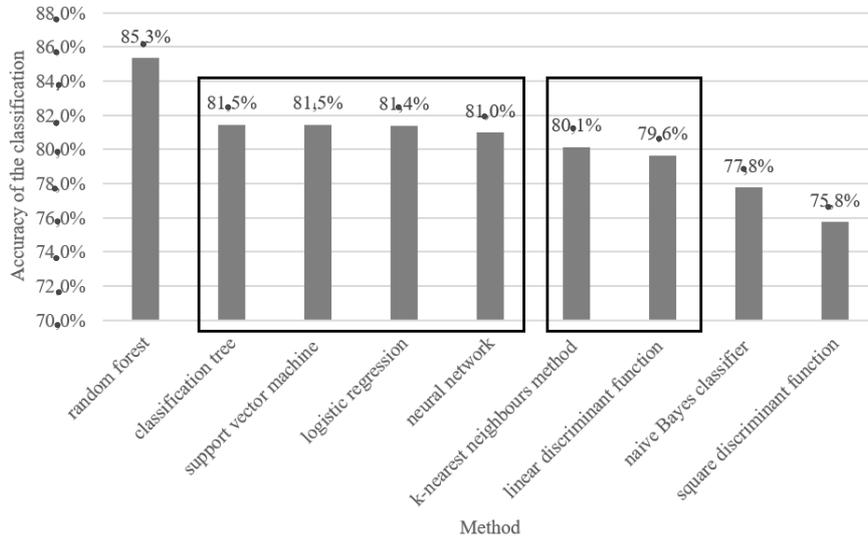
- a) the method for the selection of prognostic variables: *tstatistics* was used,
- b) a forecast timescale of one year was adopted,
- c) the prediction error was estimated for 100 samples with sizes: 40, 70 and 100 companies.

Graphs 4–6 show the average level of accuracy of classification for the methods of forecasting the insolvencies of enterprises, taking into account the different sizes of samples drawn. Again, in order to verify whether this average level is the same for all methods, a single-way analysis of variance was used. The results of the test indicate that there are grounds to reject such a hypothesis. The Tukey test was re-used to determine which methods of forecasting the failure of companies have different accuracy classifications. As at the first stage of the study, the methods in which this difference is not statistically significant were framed.

The analysis of the graphs allows the statement that regardless of the size of the research sample, the method of random forests is the one among all the methods analysed that allows the highest accuracy of classification to be obtained. The least efficient methods—characterised by the lowest average accuracy of classification—proved to be each time the square discriminant function and the naive Bayes classifier. The analysis of the Tukey test results (which are reflected in the frames on the graphs) allows us to state that as the size of the test sample decreases, the differences in the accuracy of the classification of the tested methods decrease. In the extreme case—for research samples of 40 companies—there is no statistically significant difference between the average level of accuracy of classification for as many as 7 of the 9 methods of forecasting insolvencies of enterprises. On the basis of the results obtained, it can also be concluded that the accuracy of classification of all methods surveyed is lowest in the case of the smallest samples (40 companies).

**Graph 4**

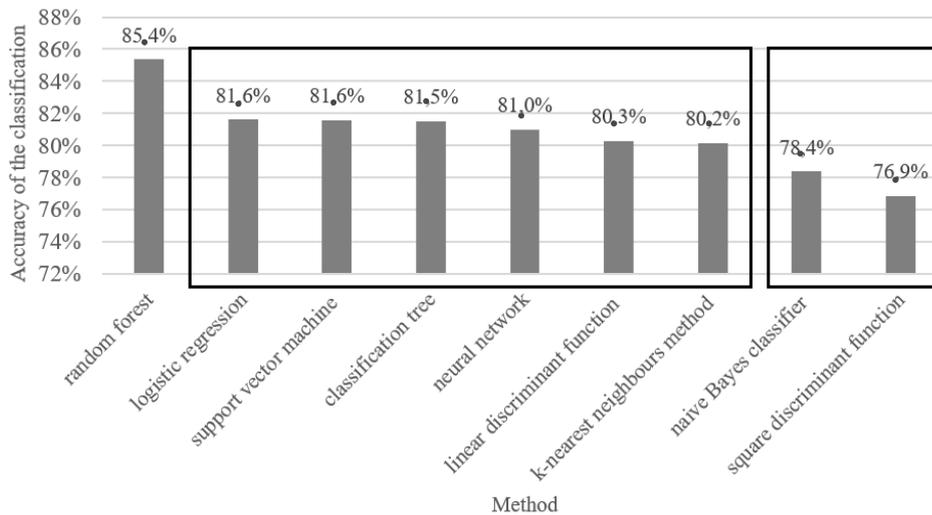
Accuracy of the classification of the business failure forecasting methods studied—the size of the samples randomly drawn: 100 companies



Source: author's own studies.

**Graph 5**

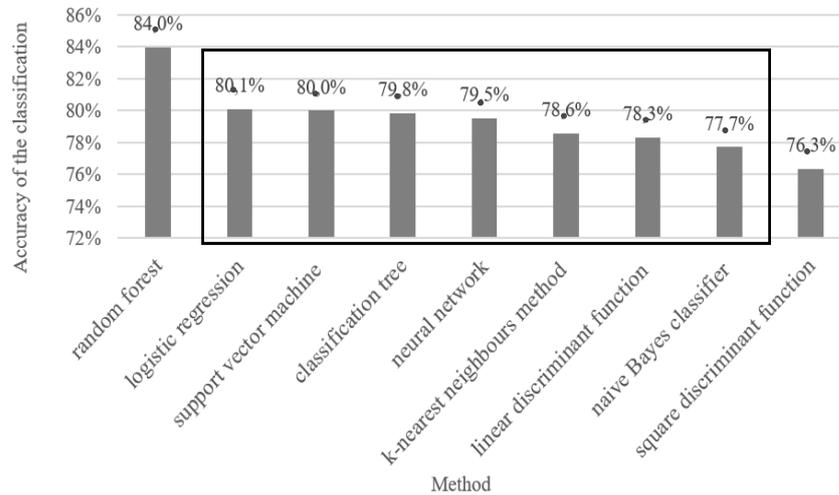
Accuracy of the classification of the business failure forecasting methods studied—the size of samples randomly drawn: 70 companies



Source: author's own studies.

**Graph 6**

Accuracy of the classification of the business failure forecasting methods studied—the size of the samples randomly drawn: 40 companies



Source: author' own studies.

At the last, third stage of the analysis, there was an examination of whether the accuracy of the classification, and thus the ranking of the methods of forecasting the failure of businesses, depends on the period for which the forecast is made. To this end, the following assumptions were made:

- the method for the selection of prognostic variables: *tstatistics*, was used
- the prediction error was estimated for 100 randomly drawn sub-samples of the size of 100 companies,
- different forecast timescales were adopted. For this purpose, the financial ratios identifying the financial condition of the enterprise that were used were made for the following periods:
  - one year prior to the filing of an application for insolvency,
  - two years before the filing of an application for insolvency,
  - three years before the filing of an application for insolvency.

Table 2 shows how the average level of accuracy coefficients of the classification of the methods examined was changing, taking into account the different length of the forecast horizon. The results presented allow us to state that as the period for which the forecast is made increases, the average accuracy of the classification of all the surveyed methods decreases. The smallest decrease in this predictive capacity was observed in the case of linear discriminant analysis (a decrease by 15.5 percentage points). The analysis of the ranking positions of individual research methods allows us to state that the accuracy of classification depends to a large extent on the time scale of the forecast.

**Table 2**

Accuracy of the classification of the business failure forecasting methods studied – depending on the timescale of the forecast

The forecasting method	Timescale					
	One year		Two years		Three years	
	Accuracy of classification	Ranking	Accuracy of classification	Ranking	Accuracy of classification	Ranking
Random forest	85.3%	1	72.8%	1	63.9%	4
Classification tree	81.5%	2	68.3%	5	59.6%	8
Support vector machine	81.5%	3	69.9%	3	64.1%	2
Logistic regression	81.4%	4	70.7%	2	64.2%	1
Neural network	81.0%	5	67.6%	6	60.5%	6
<i>K</i> -nearest neighbours	80.1%	6	66.1%	8	60.1%	7
Linear discriminant function	79.6%	7	69.8%	4	64.1%	3
Naive Bayes classifier	77.8%	8	66.5%	7	61.7%	5
Square discriminant function	75.8%	9	63.7%	9	58.7%	9

Source: author's own studies.

Only the method of random forests, regardless of the period for which the forecast is made, is always among the most efficient techniques used for this purpose.<sup>29</sup> As the forecasting period is extended, the linear discriminant

<sup>29</sup> According to the Tukey test results, there is no statistically significant difference between the accuracy of the classification of the four most efficient methods of forecasting bankruptcies in the three-year forecast time horizon.

analysis, which is very popular among practitioners, occupies an increasingly high position in the ranking of the methods studied.

## V. CONCLUSIONS

The article presents a review of Polish empirical research, in which attempts were made to use various methods of forecasting the insolvency of companies. Owing to their narrow range, as well as to the research methodology adopted, a comprehensive comparative analysis of various methods of forecasting the failure of enterprises was made. The methods were compared according to the level of accuracy of classification that can be achieved using models constructed by the use of these methods.

The results of the empirical survey provided several interesting conclusions. Firstly, the ranking of the surveyed methods of forecasting the insolvency of enterprises does not depend on the predictive variables selection method used. The situation is different in the case of sample size. The results obtained indicate that the size of the surveyed group of enterprises influences the accuracy of classification of models constructed on the basis of various forecasting methods. In the case of small research samples, the differences between the predictive capacity of different methods of forecasting insolvencies cease to be statistically significantly different. The results of empirical analysis also clearly indicate that the predictive capacity of the methods analysed is strongly influenced by the length of the period for which the forecast is made. This aspect must therefore be taken into account in the design of the relevant model. The last, very practical conclusion that may be drawn from the results of the study is that regardless of the method of predictive variables selection, sample size and forecast horizon, the random forest method has the highest predictive capacity of all methods compared. It should be remembered that these conclusions were drawn on the basis of the sample of 180 public limited liability companies and therefore they should not be attributed universality features.

The author believes that the empirical analysis should be extended in future research. An interesting area, recently very popular, is that of hybrid classification models. They have been used in research by, among others, Chuang.<sup>30</sup> It would be worth checking whether their complex character is reflected in higher predictive capacity.

*Sergiusz Herman*  
*Poznań University of Economics and Business*  
*sergiusz.herman@ue.poznan.pl*

---

<sup>30</sup> C.L. Chuang, Application of hybrid case-based reasoning for enhanced performance in bankruptcy prediction, *Information Sciences* 236, 2013: 174–185.

A COMPARATIVE ANALYSIS OF THE PREDICTABILITY OF SELECTED METHODS  
FOR PREDICTING BUSINESS FAILURE

S u m m a r y

Business failure is a feature of any developed market economy. This phenomenon entails high costs, both economic and social. For this reason, attempts have been made continuously since the beginning of the twentieth century to predict failures of businesses. The interest in this issue is reflected in the application of increasingly advanced statistical methods. The aim of the paper is to compare the predictive capacity of nine methods used in the literature to predict the bankruptcy of enterprises. The empirical research was conducted on the basis of the financial data of 180 Polish public limited companies. Its results made it possible to state that the accuracy of classification of particular methods (and thus their rating) depends on the size of the research sample and on the length of the forecast period. It was also found that the rating of the tested methods tested does not depend on the method of selection of predictive variables chosen.