RUCH PRAWNICZY, EKONOMICZNY I SOCJOLOGICZNY
Rok LXXXVII – zeszyt 2 – 2025
https://doi.org/10.14746/rpeis.2025.87.2.14

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COMPARATIVE ANALYSIS OF THE INTERRELATIONS BETWEEN THE STOCK, GOLD, AND CRUDE OIL MARKETS: EVIDENCE FROM THE WEIMAR TRIANGLE COUNTRIES

ANALIZA PORÓWNAWCZA ZALEŻNOŚCI POMIĘDZY RYNKAMI AKCJI, ZŁOTA I ROPY NAFTOWEJ NA PRZYKŁADZIE KRAJÓW TRÓJKĄTA WEIMARSKIEGO

The paper aims to investigate and compare the relationships between the stock, gold, and crude oil markets as key financial assets in the Weimar Triangle countries. The markets of the Weimar Triangle member countries have not been previously considered and compared as a group of cooperating countries. The study covers a period of heightened volatility on commodity markets (gold, crude oil) due to the COVID-19 pandemic and the outbreak of the war in Ukraine. The methodology applied includes stationarity testing followed by the estimation of a vector error correction model to examine cointegration between asset prices and a test for the Granger causality in France, Germany, and Poland. The main findings indicate that stock returns Granger-cause crude oil returns in France, Germany, and Poland. However, a statistically significant correlation is observed only in France. Moreover, gold returns Granger-cause stock returns only in Poland, despite significant correlations in all countries. In contrast, the gold and the crude oil markets are independent. In the case of observed Granger causality, asset prices can be predicted based on the evolution of the prices of the other analysed assets. There exists cointegration of asset prices, with gold prices moving in the same direction as stock prices, and oil prices in the opposite direction in France and Germany, while in Poland the relationships are reversed. Cointegration implies that the prices of the analysed assets exhibit a tendency to return to equilibrium in the long run.

Keywords: Weimar Triangle; Granger causality and cointegration; stock; gold; crude oil
JEL: G11, G15, C32

Celem artykułu jest zbadanie i porównanie relacji pomiędzy rynkami akcji, złota i ropy naftowej jako głównych klas aktywów finansowych w krajach Trójkąta Weimarskiego. Rynki krajów będących członkami Trójkąta Weimarskiego nie były dotychczas w literaturze rozpatrywane i porównywane jako grupa krajów współpracujących. Badanie uwzględnia okres większej zmienności na rynkach surowców (złoto, ropa naftowa) w związku z COVID-19 i wybuchem wojny na Ukrainie. Zastosowana metodologia obejmuje testowanie stacjonarności, a następnie estymację wektorowe-

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RPEiS 87(2), 2025: 243–258. *ISSN (Online) 2543-9170 ISSN (Print) 0035-9629*

go modelu korekty błędem w celu zbadania kointegracji cen aktywów oraz testowanie przyczynowości w sensie Grangera we Francji, Niemczech i Polsce. Główne wynik badań wskazują, że stopy zwrotu z akcji są przyczyną w sensie Grangera dla zwrotów z inwestycji w ropę naftową we Francji, Niemczech i Polsce. Jednakże istotną statystycznie korelację wykazano jedynie we Francji. Co więcej, stopy zwrotu ze złota są w sensie Grangera przyczyną rentowności akcji tylko w Polsce, pomimo istotnych współczynników korelacji we wszystkich krajach. Natomiast rynki złota i ropy naftowej są niezależne. W przypadku zaobserwowanej przyczynowości Grangera można prognozować ceny aktywów na podstawie ewolucji cen pozostałych wymienionych aktywów. Występuje kointegracja cen aktywów, przy czym kierunek zmian cen złota jest zbieżny, a dla cen ropy naftowej odwrotny w porównaniu z trendami cen akcji we Francji i Niemczech, podczas gdy w Polsce zależności są odwrotne. Kointergacja oznacza, że ceny badanych aktywów wykazują tendencję do powrotu do równowagi w długim okresie.

Słowa kluczowe: Trójkąt Weimarski; przyczynowość w sensie Grangera i kointegracja; akcje; złoto; ropa naftowa
JEL: G11, G15, C32

I. INTRODUCTION

The assets under investigation are widely perceived as leading investment options in the financial market. Stocks and gold are usually considered alternative assets due to their low or negative correlation. The low returns observed during the recent crises affecting the stock markets have made gold, regarded as a relatively safe investment, more attractive to investors. The causality running from crude oil to stock and gold markets is also documented in the literature. It is observed that rising commodity prices lead to higher production costs and falling stock prices. In contrast, gold and crude oil are positively correlated, which can be explained by cost inflation. The rise in oil prices impacts the demand for gold, which in turn becomes more expensive. In this case gold plays the role of a hedge against inflation (Beckmann et al., 2015; Mamcarz, 2018; Singh, 2014; Tursoy & Faisal, 2018).

The aim of this paper is to investigate and compare the relationships between the stock, gold, and crude oil markets as key financial assets in the Weimar Triangle countries. Although these assets are widely analysed in the literature, less attention has been paid to Europe, especially to the three countries analysed in my paper. Moreover, the higher volatility in commodity markets (gold, crude oil) caused by the COVID-19 pandemic and the outbreak of the war in Ukraine may have an impact on stock markets in France, Germany, and Poland.

The analysed countries form the Weimar Triangle. Recently, the Weimar Triangle, which for years had been an important tool of economic and political integration between the two Western European leaders and Poland, has been renewed. The interrelations between markets in these countries should be examined as closer cooperation may translate into increased investor interest in these markets. I aim to evaluate whether the countries cooperating on polit-

ical and economic levels show similar causal patterns regarding financial markets among themselves and compared to the mainstream financial markets.

To my knowledge, the European countries analysed in my study have not previously been considered and compared as a group of cooperating nations. The originality and novelty of the study lie in selecting and analysing this particular group of countries under the circumstances mentioned above. It is assumed that the analysed markets are cointegrated, and that the direction of causality runs from the stock to the gold market, while the crude oil market Granger-causes the stock market. Causality between the considered assets is also hypothesized to exist.

The research method involves the application of the vector error correction model (VECM), which is a valuable tool for investigating interactions among economic and financial variables and the dynamic behaviour of the financial time-series data, that is, analysing the existence of the long-term equilibrium between the considered markets and testing for Granger causality in the short-term (Kusiedł, 2000, pp. 48–50; Wu & Zhou, 2015). To track asset price fluctuations, I utilize the stock exchange indices of shares listed on the Euronext Paris (CAC40), the Frankfurt Stock Exchange (DAX), and the Warsaw Stock Exchange (WIG20); in the case of commodities, I use the gold spot prices London Metal Exchange (LME) and the Europe Brent Spot Price (FOB), expressed in domestic currencies.

The paper is organized as follows: section II provides a literature review, section III presents the methodology applied, including a description of the non-stationarity test, the model specification and the post-estimation tests. In section IV, the empirical data are characterized. Section V presents the empirical results, namely the analysis of stationarity and cointegration of the variables and Granger causality relations, and section VI consists of the discussion and conclusions.

II. LITERATURE REVIEW

The relationship between gold, crude oil, and stocks is the subject of many studies. Such research applies various methods to investigate the existence of cointegration and Granger causality. The research tends to focus on geographical regions, especially Asian, African and American countries. In general, less attention is paid to European economies. The main selection criteria are the stock markets of leading economies and the inclusion in the group of the key producers or consumers of the analysed commodities. Below we present a review of the research that matches the above criteria.

For instance, Hussin et al. (2013), in their study on Asia, analysed causality and cointegration between crude oil, gold, and stock prices in Malaysia from January 2007 to December 2011. According to their results, stock returns are not the Granger cause of gold returns, and no cointegration between Islamic stock markets and strategic commodities in the long run was observed. The

linkages between gold, oil prices, and the Islamic stock market was also examined by Chkili (2022), who applied the standard VAR and Markov switching VAR models. The data span covered the years 1996–2020, including the recent COVID-19 pandemic crisis. Based on VAR estimates, Chkili indicates that the relationship between crude oil and Islamic stock markets is positive, as opposed to the negative relationship between the Islamic stock market and gold. A study by Bouri et al. (2017) concerning the correlations between global markets of gold, oil and Indian stocks between June 2009 and May 2016 showed a positive influence of the implied volatility of gold and oil prices on the stock market and a reverse bidirectional causality between the implied volatility of gold prices and oil prices. In contrast, Tursoy & Faisal (2018), based on the ARDL model, showed, on the example of Turkey, a negative correlation between gold prices and stock prices, while a correlation between oil prices and stock prices was positive. The results of the Granger causality test also confirm the unidirectional causality between the gold price and the stock price in the period between January 1986 and November 2016. The observed patterns differ from the results concerning India or Malaysia.

Attention is frequently paid to China as a leading world economy. Wu and Mai (2024) investigated the crude oil and gold markets, alongside the Chinese stock market, including the CSI 300 sector indices. Based on the generalized vector autoregression (GVAR), they prove the asymmetric spillovers between commodities (gold, crude oil) and the sectors under analysis from 2000 to 2023. The results vary depending on sector analysed. The spillover effects of asymmetry are mainly attributed to the global financial crisis (GFC), the European sovereign debt crisis (ESDC), the decline in oil prices, and the outbreak of the COVID-19 pandemic.

A global perspective is a commonly proposed approach. Mensi et al. (2021), concentrated on the US and China as world leaders. While examining the spillovers between the stock, crude oil, and gold futures markets before and during the COVID-19 pandemic, they showed that the spillovers from commodity markets to the US and Chinese stock markets became more intense due to the COVID-19 pandemic, which acted as a major public health and economic shock. The authors stressed that gold, as in the past during periods of financial and energy crises, once again appears to be a strong safe haven. The interactions between the aforementioned markets were also investigated by Arfaoui and Ben Rejeb (2017), who confirm a significant relationship between all the markets considered. The results indicate a negative relation between oil and stock prices, while oil price was positively affected by gold and USD. The impact of oil futures prices and Chinese oil gross imports on oil price was also reported. Also, the changes in oil, USD, and stock markets were found to have driven gold fluctuations. Additionally, they indicated the influence of these markets on the US dollar.

The United States was also analysed separately by researchers, for example, Gokmenoglu and Fazlollahi (2015) showed the existence of bilateral causality between changes in the gold price and S&P500 for a period lasting from January 2013 to November 2014. They confirmed the evidence of

unidirectional causality from the oil price to the stock index S&P500. Moreover, changes in the oil price were found to Granger-cause changes in the gold price. Further research, conducted by Hung and Vo (2021), also focused on the US. They used the spillover index and the wavelet coherence approaches to assess the co-movements between WTI, gold and S&P 500 before and during the COVID-19 outbreak, covering data from 1 January 2018 to 30 January 2020. The analysis carried out across subperiods revealed the time-varying dynamic spillovers with different intensities and directions of transmission during the COVID-19 outbreak. Stocks and crude oil are net receivers of shocks, while gold was a net transmitter of risks before the COVID-19 period. During the COVID-19 outbreak, the transmitting role was assigned to crude oil and S&P 500 markets, while gold became the net recipient of the return spillovers.

Analyses devoted to Latin America can be exemplified by Mexico (Singhal et al., 2019). In this case, the ARDL approach was adopted to examine the long-term dynamic relations among international oil and gold prices, the exchange rate, and the stock market index. They showed that time series are cointegrated. Moreover, gold prices positively determined the stock price in Mexico and did not significantly affect the exchange rates. On the other hand, crude oil prices negatively affected stock prices and exchange rates.

Another criterion for selecting countries included in the analysis is membership in the group of oil-exporting and oil-importing countries, which was adopted, for example, by Dontoh et al. (2024). The economies included in this study were Nigeria, Ghana, Egypt, Algeria and Tunisia. The authors used quantile regressions (QR) and quantile-on-quantile regression (QQR). Their study, lasting from 1 January 2017 to 30 September 2021, included the impact of the COVID-19 pandemic. According to the results, the role of crude oil depended on the countries and the market conditions. It served as a hedge, diversifier or safe haven against stocks. Also, the role of gold differed over the years. The interrelations in the Middle East oil-exporting countries (Bahrain, United Arab Emirates Kuwait, Qatar, and Saudi Arabia) and oil-importing countries, including Jordan, Egypt, and Turkey, were investigated by Bani-Khalaf and Taspinar (2022). They also confirmed the existence of dependences during the COVID-19 pandemic.

In the case of Europe, I evaluated the relationship between the joint Central European stocks index CETOP, crude oil, and gold in the period 2002–2021. The results indicated that gold returns Granger-caused stock returns, and stock returns, in turn, determined crude oil returns. The reverse relationship between gold and stock prices was reported, while the stock and the crude oil markets evolved in the same direction (Mamcarz, 2024). Another study, conducted by Wanat et al. (2015) also analysed linkages between these commodities and stocks (DAX, FTSE100 indices) based on empirical copula. The findings show the time-varying dependences and the lack of causality running from commodity prices to the stock markets analysed, whereas gold was dependent on stock markets in the most considered sub-periods between 2 January 1998 and 30 June 2014.

The criteria presented above are commonly used by researchers. However, other options are also possible, such as that adopted in my study, namely, membership in political and economic associations, leading to closer cooperation between countries and drawing the attention of investors who are looking for ways to invest capital, especially in turbulent times.

III. RESEARCH METHODOLOGY

In my study, I use end-of-month data on commodity prices and stock market indices, which constituted the basis for calculating the monthly logarithmic rates of return, defined as the first differences of the above-mentioned financial time series data expressed in logarithms. I began by analysing the time series features applied when investigating the interrelations between stock, gold, and crude oil markets in France, Germany, and Poland associated with the Weimar Triangle. To indicate the order of logarithmic asset prices integration, the Augmented Dickey-Fuller Unit Root (ADF) Test (Dickey & Fuller, 1979), DF-GLS Test (Elliott, Rothenberg, & Stock, 1996), and the Phillips-Perron Test (Phillips & Perron, 1988) were applied. The differentiation of the time series made it possible to obtain stationary variables, which were prerequisites for estimating the VAR/VECM model. As the first order of time series integration was determined, I took the first differences of logarithms as my variables in the short-term approach. At the same time, the long-run dependencies were analysed based on the logarithmic asset prices.

The causal relationships between stock, gold, and crude oil markets were examined based on the estimates of the three-dimensional VAR(k). The procedure was proposed by Granger (1969). I used the Wald Test to check zero constraints on the estimated VAR(k) coefficients for each equation separately (Lütkepohl, 2005, pp. 45; 102–103). I independently estimated a VAR(k) model for each country (j): France, Germany, and Poland, formulated in the general VECM form (cf. Juselius, 2006, pp. 61–63):

$$\Delta x_{j,t} = \Gamma_{j,1}^{(m)} \Delta x_{j,t-1} + \Gamma_{j,2}^{(m)} \Delta x_{j,t-2} + \cdots + \Gamma_{j,k-1}^{(m)} \Delta x_{j,t-k+1} + \Delta x_{j,t-1} + \Pi_j x_{j,t-m} + \Phi_j D_{j,t} + \varepsilon_{j,t}, \quad (1)$$

where: $\Delta x_{j,t}$ – $p \times 1$ dimensional vector of variables in the first differences, namely logarithmic asset returns (D.ln_ith asset) for jth country; $x_{j,t-m}$ – $p \times 1$ dimensional vector of logarithmic asset prices (ln_ith asset) for jth country; $\Gamma_{j,k-1}^{(m)}$ – coefficients matrix of k-lagged logarithmic returns for jth country; Π_j – coefficients matrix of m-lagged logarithmic prices for jth country, $\Pi = \propto \beta$; β_j – coefficients matrix of the cointegrating vectors, reflecting the long-term impacts for jth country; α_j – coefficients matrix of the speed of adjustment to long-term equilibrium, reflecting the short-term impacts for the jth country; $D_{j,t}$ – vector of deterministic components for jth country; Φ_j – coefficients matrix of vector D_t; k – lag length of the VAR model for the jth country; m – lag of the ECM term for the jth country.

In the case of non-stationary variables of vector x_t,the selection of the p-variable model form depends on the number of linearly independent cointegrating relationships among the logarithmic prices. It was essential to establish the rank r of the Π matrix, such as $0 \leq r < p$ to investigate if there was a long-term equilibrium between these non-stationary times series. I tested the null hypothesis of the Johansen test, stating that there is no cointegration vector (H0: $r = 0$), against an alternative hypothesis (H1: $r \leq 1$). If the null hypothesis was rejected, the procedure was continued for higher ranks ($r \leq p$) until the number of cointegrating vectors in the VECM model was established. If there was no cointegration relationship between logarithmic prices x_t), $r = 0$. Thus, the Πx_{t-m} also equaled 0, suggesting the error correction mechanism (ECM) omission in the VECM. This means that model (1) was estimated exclusively based on the first differences, namely the logarithmic asset returns (Becketti, 2013, p. 390; Charemza & Deadman, 1997, pp. 129, 175; Johansen & Juselius, 1990).

I also consider that the specification of the VECM model depended on the trend type, the number of lags, and the restrictions imposed. I analysed five cases as follows: Unrestricted trend (trend), Restricted trend (rtrend), Unrestricted constant (constant), Restricted constant (rconstant), and No trend (none) involving the inclusion of the linear trend and quadratic trend parameters in $x_{j,t}$; deterministic terms in the cointegrating equation representing the means and linear trends of this relationship, respectively. The general assumptions for trend specification were presented by Becketti (2013, p. 392).

The post-estimation procedure included the study of the autocorrelations and normality of the residuals of estimated VECM. I applied the Lagrange-multiplier (LM) Test for residual autocorrelation, which is particularly useful for detecting low-order autocorrelations. The null hypothesis (H0) assumed no autocorrelation at lag order. Testing for residuals nonnormality involved carrying out the Jarque-Bera test (Lütkepohl, 2005, pp. 345–348; 174–177). At the same time, the structural breaks of residuals were tested by applying the cumulated sum of the forecast errors test, CUSUM (Enders, 2010, pp. 106–107). I used Stata software and MS Excel for computation in this study.

IV. THE CHARACTERISTICS OF THE EMPIRICAL DATA

To describe the financial markets of the Weimar Triangle, I used the following stock indices as proxies: CAC40, DAX, and WIG20. The CAC40 index is the most widely recognized benchmark of the French stock market, tracking the performance of the forty most significant shares listed on Euronext Paris; it is a free-float market capitalization-weighted index (Euronext, 2024). The DAX index reflects the performance of the forty most extensive German stocks with the highest turnover listed on the Frankfurt Stock Exchange Regulated

Market (Frankfurt Stock Exchange, 2024). Finally, the WIG20 consists of the twenty biggest and the most liquid companies, called blue-chips, listed on the GPW Main List (GPW Benchmark, 2024). The other two assets were gold and Brent crude oil, whose prices were expressed in EUR in the case of the two considered eurozone countries or PLN for Poland.

I used the exchange rates to calculate these prices in the domestic currencies of the Weimar countries based on the gold spot prices (London Fix) and the Europe Brent Spot Price (FOB), both with monthly frequency. Data and information were retrieved from the dedicated stock exchange websites: Euronext (2024), Frankfurt Stock Exchange (2024), GPW Benchmark (2024), financial portal Stooq (2024a, 2024b, 2024c), WGC (2024), NBP (2024) and EIA (2024). The sample covers the period from January 1999 to April 2024 (end-of-month), including 304 observations. All values were transformed into logarithms before conducting further analysis and estimations. I considered the logarithmic prices (variables at levels) and logarithmic rates of returns on investment in the analysed assets (variables at first differences). The price patterns of the analysed financial assets are depicted in Figures 1 (stocks) and 2 (gold and crude oil), while the descriptive statistics are presented for France, and Germany in Table A1, and for Poland in Table A2 of the Appendix to the paper.[1]

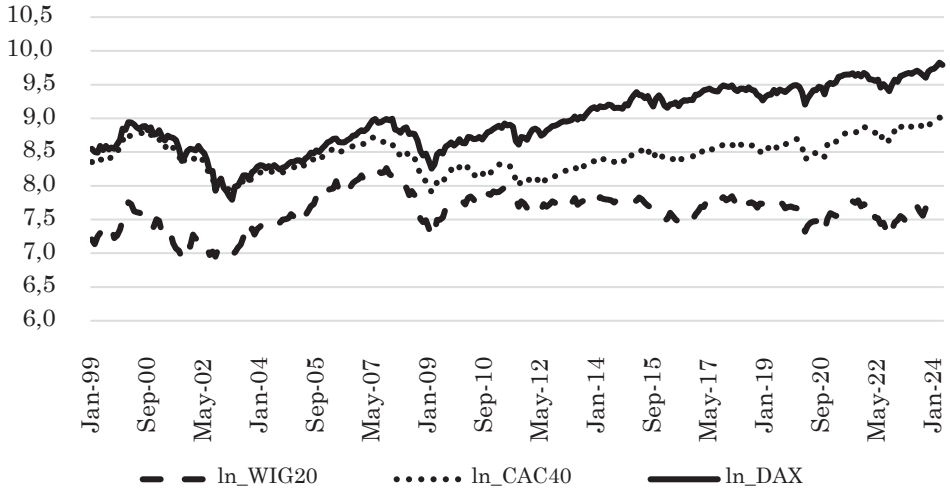
The stock indices showed similar trends over time. I observed periods of both rising and falling prices. However, the DAX index was more volatile in terms of standard deviation and coefficient of variation. Only the CAC40 index exhibited a right-skewed distribution, while the other two were skewed to the left. Nonetheless, the values of these statistics indicate that the data were pretty symmetric. CAC40 and DAX were platykurtic, while WIG20 was almost mesokurtic. Logarithmic gold and crude oil prices fluctuated more than the stock market benchmarks. Although the skewness statistics indicated the left-skewed distribution, the crude oil price appeared moderately skewed, while gold was still reasonably symmetrical. Gold prices were platykurtic, while crude oil prices were nearly mesokurtic. These features are characteristic of prices expressed in both currencies.

Concerning the first differences, defined as the logarithmic returns, the highest fluctuations among stock indices were attributed to the WIG20. Gold returns were the least volatile, while oil returns were almost as volatile as the DAX index. In France and Germany, all asset returns were left-skewed, except for gold, which was slightly right-skewed. In the other cases, the skewness varied from moderate to high depending on the returns analysed. For the Polish capital market, the stock returns were slightly left-skewed, while the gold returns showed a right-skewed distribution. However, the low skewness coefficients suggest that data were almost symmetrical. At the same time, crude oil returns were left-skewed with moderate deviation from the normal distribution. Moreover, in three countries the logarithmic stock, gold and crude oil returns, regardless of the currency applied, were leptokurtic.

[1] The Appendix to the paper is available in the OSF repository: https://osf.io/bdcv7/

Figure 1

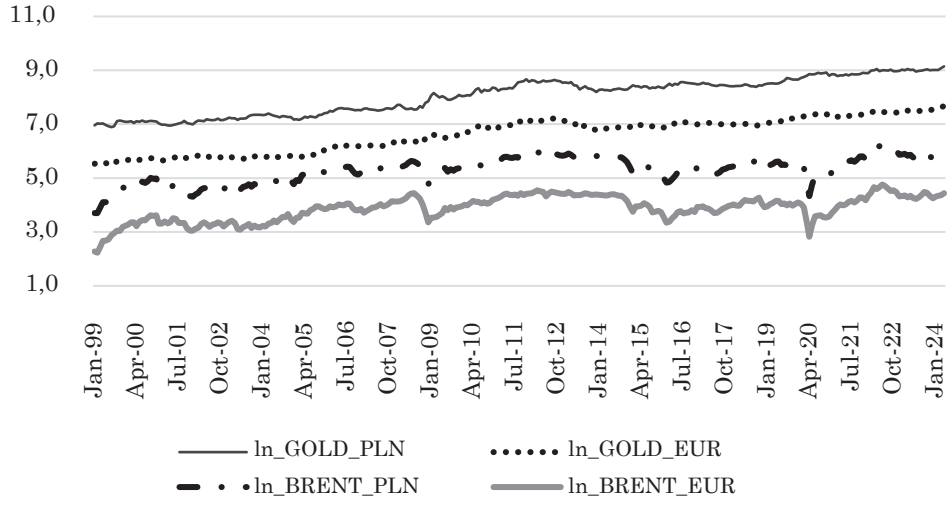
The logarithmic values of the stock indices WIG20, CAC40, and DAX
in the period January 1999 – April 2024 (end-of-month)



Source: the author's elaboration based on Stooq (2024a, 20024b, 2024c).

Figure 2

The logarithmic prices of gold and crude oil in EUR and PLN
in the period January 1999 – April 2024 (end-of-month data)



Source: the author's elaboration based on EIA(2024); NBP (2024); WGC (2024).

The results of the selected unit root tests, namely the ADF, DF-GLS, and Phillips-Perron test, are presented in Tables A3, A4, and A5 of the Appendix, respectively. In the case of the ADF test, I accept the null hypothesis of a unit root for all analysed variables at standard significance levels (1%, 5%, 10%) except for crude oil log-prices (BRENT_EUR, BRENT_PLN at level) which were significant in at 5% and 10%, but not at 1%. The DF-GLS test results also confirm that all variables at levels are nonstationary regardless of the number of lags and significance levels. Moreover, for both Phillips-Perron test statistics, the null hypothesis of the unit root was accepted except for BRENT_EUR and BRENT_PLN, for which the null should be rejected at 10% and 5% but not at 1%. The first differences of the analysed variables seem to be stationary processes as I rejected the null hypothesis of the ADF test, DF-GLS, and Phillips-Perron at any significance level.

After investigating the time series stationarity based on the conducted unit root tests, I concluded that the logarithmic variables (at levels) were non-stationary. In contrast, their first differences, defined as the logarithmic returns, were stationary. This means that logarithmic prices are integrated of order one. The presented features of the analysed time-series justify conducting cointegration analysis between financial asset prices in the considered markets before the estimation process.

Since I confirmed the stationarity of the first differences of logarithmic asset prices, defined as the logarithmic returns, I was able to calculate the correlation coefficients between obtained rates of return from investments in the analysed markets. In this way, I addressed the problem of the spurious relationships between variables, which could exist in the case of asset prices. The results are presented in the Appendix, Table A6.

I observed a very strong positive correlation between the stock returns in France and Germany. In contrast, fairly strong positive relationships existed between Poland and the other two markets. All the coefficients were statistically significant at 1%. The gold returns based on EUR and PLN prices were strongly positively correlated. The crude oil returns showed the same pattern. The significant relationship between stock and gold returns was a weak negative one in the case of considered eurozone countries and a negative moderate one in the case of Poland when assets were priced in the domestic currencies. Only in the case of France was a very weak positive correlation between stock and crude oil returns indicated.

V. RESULTS

This section presents the characteristics of the three country-specific models separately. The results are provided in the tables of the Appendix. I start with the French market (Table A7–A9); next, I address the German case (Table A10–A12), followed by the Polish one (Table A13–A15). The investigation of the optimal number of lags was conducted to choose the proper specification

of the Johansen test for cointegration and for the subsequent model estimation process for each country.

While analysing the results in Table A7, I conclude that the optimal number of lags in the French market is two, with one exception. Based on the results of the cointegration analysis summarized in Table A8, I considered one cointegration vector and two lags in the final VAR model representation based on Max and AIC. Moreover, after comparing different model specifications for the French market, I decided to eliminate the trend term in the cointegration relationship as I accepted the null that an unrestricted constant model (constantlags2) is nested in a restricted trend model (rtrendlags2) at 5% (Prob > chi2 = 0.0794). However, I could no longer trim the unrestricted constant model (constantlags2) since the null that the restricted constant model (rconstantlags2) is nested in the unrestricted constant model (constantlags2) was rejected at 1% (Prob > chi2 = 0.0074).

The characteristics of the selected model (constantlag2) are summarized in Table A9. For the short-run equations, I observed a statistically significant coefficient of the error correction mechanism in the equations describing the returns on the stock market (D.ln_CAC40) and the crude oil market (D.ln_BRENT_EUR), indicating the ability of the system to return to long-term equilibrium. Moreover, the crude oil returns depended on their previous values and the changes in stock prices. At the same time, gold returns were explained by their own lagged values. All coefficients were statistically significant in the long-term equation of VECM, called ECM (ce1). The evolution of the gold prices and the stock prices in the same direction is observed, while the crude oil prices moved in the opposite direction to the stock prices.

Germany was the second country I examined. As presented in Table A10, the optimal number of lags in most cases is two. I included the two lags in the VAR representation while conducting the cointegration analysis, the results of which are summarized in Table A11.

The statistics presented in Table A11 suggested one cointegration vector except for the SBIC criterion under assumed unrestricted constant trend specification. I chose this type as I accepted the null that an unrestricted constant model (constantlags2) is nested in a restricted trend model (rtrendlags2) at 5% (Prob > chi2 = 0.1674). The null hypothesis that the unrestricted constant model (rconstantlags2) is nested in the unrestricted constant model (constantlags2) was rejected at 5% (Prob > chi2 = 0.0048).

The selected model specification is presented in Table A12. I observed that the coefficients assigned to the ECM (ce1) vector were statistically significant in the case of DAX and BRENT equations, allowing the system to return to long-term equilibrium. Gold returns (equation GOLD) were only explained by their lagged values and the constant term, while the other parameters were not statistically different from zero. Crude oil returns (equation BRENT) depended on stock and crude oil returns with one lag, as these estimates were statistically significant at 1%. For the ECM long-term relationship, I observe that gold and crude oil prices were cointegrated with stock prices. However,

the direction of influence differed, as the increase in gold price was accompanied by a rise in stock prices, while the crude oil price changed in the opposite direction compared to the DAX index values.

Results were also obtained for the Polish capital market. I began with the number of lags, two or one, depending on the criterion (Appendix, Table A13).

After comparing different model specifications for the Polish market, I selected the restricted constant specification based on nested models testing. The assumption that the unrestricted constant model (constantlags2) is nested in the restricted trend model (rtrendlags2) was accepted at 5% (Prob > chi2 = 0.0690). However, the unrestricted constant model (constantlags2) was not nested in the restricted constant model (rconstantlags2) since Prob > chi2 = 0.0330 is less than 0.05. The cointegration test indicated one cointegrating relationship between stock, gold, and crude oil prices in Poland for the specifications considered (Appendix, Table A14). The estimates of the unrestricted constant model are presented in Table A15 of the Appendix.

Concerning the short-term equations, stock returns were explained by gold returns at a 10% significance level. In contrast, all coefficients in the gold return equation, except for the intercept term, were statistically insignificant. Brent returns depended on lagged stock and crude oil returns, as well as the ECM term (p-value < 1%). Although the ECM vector estimates were statistically significant, the ability to return to long-term equilibrium was proved only in the case of the Brent returns equation. The fall in the gold price was accompanied by an increase in the stock price, while the drop in the Brent crude oil price caused a decrease in the stock price. Thus, I observed a different pattern in Poland compared to France and Germany in the long run.

A comparison of the three considered countries in terms of the Granger causality analysis based on F-test statistics for estimated coefficients of the VECM models is presented in Table A16 of the Appendix. I indicate that in the case of France, Germany, and Poland, the stock returns in these countries Granger-caused crude oil returns when calculated in their respective domestic currencies. Moreover, gold returns Granger-caused the stock returns in Poland. All the mentioned interrelations are unidirectional since the dependencies run from the stock market to gold market returns and from the stock market to the crude oil market in the indicated cases. No causal relationships between gold and crude oil were observed, indicating the independence of these markets.

Based on the results presented in the Appendix, Table A17, the Lagrange multiplier test statistics showed no evidence of residual autocorrelation at any lag order, as the null hypothesis was not rejected at 5%. However, with regard to the Jarque-Bera test, the residuals were not normally distributed except for the WIG20 equation error term. Moreover, the model's parameters were stable over time, since I could not reject the null hypothesis of the coefficient stability at 5% in the case of the CUSUM test (Appendix, Table A18).

VI. DISCUSSION AND CONCLUSIONS

When analysing the relationship between stock, gold and crude oil markets in France, Germany, and Poland, I found unidirectional Granger causality running from stock markets to crude oil markets, showing dependencies between changes in the individual stock indices and crude oil profitability. However, a statistically significant correlation coefficient was indicated for France. In contrast, Gokmenoglu and Fazlollahi (2015) highlight that the causality runs in the opposite direction, namely from the oil price to the US stock index (S&P500). My research hypothesis, which assumed the direction of the influence of crude oil on the stock market, was negatively verified for the European countries examined.

I demonstrated that the causality runs from gold returns to stock returns, but only in Poland. However, a significant Pearson correlation was observed between market returns in each country studied. My results regarding causality are consistent with those reported by Mensi et al. (2021). Moreover, they align with the study by Tursoy and Faisal (2018) who also indicate the unidirectional causality between the gold and the stock market. Nevertheless, the results contradict the findings by Hussin et al. (2013), who showed that stock prices are not the Granger-cause of gold prices in Islamic countries. In the two Weimar countries (France and Germany), unlike the USA market analysed by Bouri et al. (2017), there is no bilateral causal relationship (feedback) between changes in gold and stock prices. My results contradict the initial assumption that the causality runs from the stock to the gold market.

Additionally, contrary to the results of Gokmenoglu and Fazlollahi (2015), I found that the analysed gold and crude oil markets are independent. Moreover, no statistically significant relationships between these markets are reported in my findings regarding correlation coefficients. Also Irshad et al. (2014) showed no evidence of causality between considered financial assets.

Regarding the long-term relationship between markets, expressed by the ECM (ce1) component of the estimated models, I observed long-term equilibrium as previously assumed. However, the direction of dependences differs among the countries analysed in my study. Similar to Tursoy and Faisal (2018) and Singhal et al. (2019), I reported the same direction of movements of gold and the reverse influence of crude oil prices trends on stock prices in France and Germany, while in Poland, the dependences were the opposite. My results concerning cointegration are not always consistent with the other authors, as some of them demonstrated no ability to return to equilibrium in the long run, namely Hussin et al. (2013) and Irshad et al. (2014). The influence attributed to gold in the case of the Polish market shows that gold is perceived as an alternative asset to stocks as a rise in one asset price leads to a decrease in the other. The patterns characteristic of Poland align with the results presented in my previous study concerning the Central Europe stock markets included in the CETOP index (Mamcarz, 2024).

To some extent, the mixed findings concerning dependencies between markets in the Weimar Group countries can be attributed to the convergence gap between the Western European countries included in the Eurozone and the Central European ones, represented by Poland. Also, the use of the national currency (PLN) and EUR in the pricing of the assets analysed may cause mixed results. Moreover, unexpected events, such as the COVID-19 pandemic and the outbreak of the war in Ukraine, may have had an impact on the obtained results, as financial assets and commodity prices became more volatile in these circumstances. In the face of an unprecedented situation, relationships may change and differ from those presented in the literature. Additionally, the time horizon and the benchmark indices adopted in numerous studies may also have an influence on the results. Analysing linkages between stock, gold, and crude oil made it possible to understand the dynamics of prices and returns in the Weimar Group countries. Moreover, it can be useful in predicting reactions and volatility in the considered markets caused by turbulences in the other interrelated asset markets.

Author contributions / Indywidualny wkład autora (CRediT): **Katarzyna Mamcarz** – 100% (Conceptualization / Konceptualizacja; Formal analysis / Formalna analiza; Investigation / Przeprowadzenie badań; Writing – original draft / Pisanie – pierwszy szkic; Writing – review & editing / Pisanie – recenzja i edycja).

Conflict of interest / Konflikt interesów: The author declares no conflict of interest. / Autorka nie zgłosiła konfliktu interesów

Funding / Finansowanie: The author declares no institutional funding. / Autorka oświadczyła, że nie korzystała z finansowania instytucjonalnego.

The use of AI tools / Wykorzystanie narzędzi AI: The author declares no use of AI tools. / Autorka oświadczyła, że nie korzystała z narzędzi AI.

Data availability / Dostępność danych*:* The data is available on request. / Dane dostępne na życzenie.

Supplementary material: The Appendix to the paper is available in the OSF repository: / Aneks do artykułu jest dostępny na: https://osf.io/bdcv7/

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